**H – Quadratics, Lesson 2, Using the Discriminant (r. 2018)**

# QUADRATICS
## Using the Discriminant

| Common Core Standard | Next Generation Standard |
|---|---|
| **A-REI.4b** Solve quadratic equations by inspection (e.g., for $x^2=49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* + *bi*, *a* - *bi* for real numbers *a* and *b*.<br>PARCC: Tasks do not require students to write solutions for<br>ic equations that have roots with non-zero imaginary parts.<br>r, tasks can require the student to recognize cases in which a<br>ic equation has no real solutions. | **AI-A.REI.4b** Solve quadratic equations by:<br>i) inspection,<br>ii) taking square roots,<br>iii) factoring,<br>iv) completing the square,<br>v) the quadratic formula, and<br>**vi) graphing.**<br>**Recognize when the process yields no real solutions.**<br>(Shared standard with Algebra II)<br>Notes:<br>• **Solutions may include simplifying radicals or writing solutions in simplest radical form.**<br>• An example for inspection would be $x^2 = 49$, where a student should know that the solutions would include 7 and -7.<br>• **When utilizing the quadratic formula, there are no coefficient limits.**<br>• **The discriminant is a sufficient way to recognize when the process yields no real solutions.** |

## LEARNING OBJECTIVES

Students will be able to:

1) Identify the number and characteristics of solutions to quadratic equations based on analysis of the discriminant.

**Overview of Lesson**

| Teacher Centered Introduction | Student Centered Activities |
|---|---|
| **Overview of Lesson**<br>**- activate students' prior knowledge**<br>**- vocabulary**<br>**- learning objective(s)**<br>**- big ideas: direct instruction**<br>**- modeling** | **guided practice ←Teacher: anticipates, monitors, selects, sequences, and connects student work**<br>**- developing essential skills**<br>**- Regents exam questions**<br>**- formative assessment assignment (exit slip, explain the math, or journal entry)** |

## VOCABULARY

| | | |
|---|---|---|
| discriminant | standard form of a quadratic | root |
| real solutions | solution | x-axis intercept |
| imaginary solutions | zero | |

# BIG IDEAS

**Standard Form of a Quadratic**: $ax^2 + bx + c = 0$

**Quadratic Formula:** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

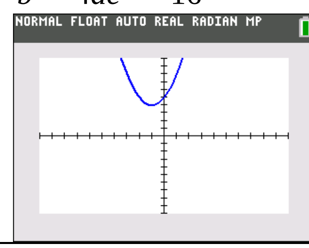
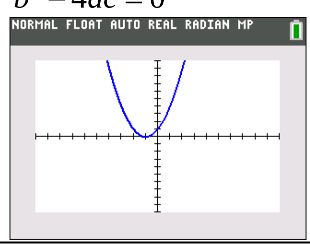
**Discriminant** = $b^2 - 4ac$

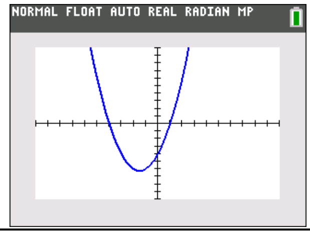
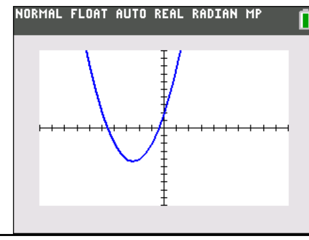
## Analyzing the Discriminant

The discriminant can be used to determine the number of and type of solutions to a quadratic equation.

Every quadratic can have zero, one, or two solutions.

Solutions can be real or imaginary numbers.

| If the Value of the Discriminant Is: | Characteristics and Number of Solutions of the Quadratic Equation Are: | Examples |
|---|---|---|
| **Negative**<br>$0 > b^2 - 4ac$ | If the value of the discriminant is negative, then there will be two imaginary number solutions and no x-axis intercepts. | $y = x^2 + 2x + 5$<br>$b^2 - 4ac = 2^2 - 4(1)(5)$<br>$b^2 - 4ac = -16$<br> |
| **Zero**<br>$0 = b^2 - 4ac$ | If the value of the discriminant is zero, then there will be one real solution and the graph will touch the x-axis at one and only one point. | $y = x^2 + 2x + 1$<br>$b^2 - 4ac = 2^2 - 4(1)(1)$<br>$b^2 - 4ac = 0$<br> |

| | | |
|---|---|---|
| **Positive**<br>Perfect Square<br>$b^2 - 4ac > 0$ | If the value of the discriminant is a positive perfect square, then there will be two integer solutions and two x-axis intercepts. | $y = x^2 + 3x - 4$<br>$b^2 - 4ac = 3^2 - 4(1)(-4)$<br>$b^2 - 4ac = 25$<br> |
| **Positive**<br>Not a Perfect Square<br>$b^2 - 4ac > 0$ | If the value of the discriminant is positive, but not a perfect square, then there will be two real number solutions and two x-axis intercepts. | $y = x^2 + 5x + 2$<br>$b^2 - 4ac = 5^2 - 4(1)(2)$<br>$b^2 - 4ac = 17$<br> |

## DEVELOPING ESSENTIAL SKILLS

Determine the number and characteristics of the following quadratic equations by analyzing the discriminant.

1. $-3n^2 + 4n + 6 = 6$
2. $-p^2 + 4p - 7 = -3$
3. $-x^2 + 5x - 3 = -3$
4. $3v^2 + 3v + 2 = 2$
5. $6v^2 - 2v + 6 = 4$
6. $p^2 - 4p - 1 = -5$
7. $-3x^2 - 2x + 4 = 4$
8. $2x^2 + 4x + 11 = 5$
9. $6x^2 + 6x + 3 = 3$
10. $3a^2 - a - 4 = -2$

Answers

1. 16; two real solutions
2. 0; one real solution
3. 25; two real solutions
4. 9; two real solutions
5. -44; two imaginary solutions
6. 0; one real solution
7. 4; two real solutions
8. -32; two imaginary solutions
9. 36; two real solutions
10. 25; two real solutions

**REGENTS EXAM QUESTIONS (through June 2018)**

# A.REI.B.4: Using the Discriminant

206) How many real solutions does the equation $x^2 - 2x + 5 = 0$ have? Justify your answer.

207) How many real-number solutions does $4x^2 + 2x + 5 = 0$ have?
1) one
2) two
3) zero
4) infinitely many

**SOLUTIONS**

206) ANS:
No Real Solutions

Strategy 1. Evaluate the discriminant $b^2 - 4ac$ for $a = 1$, $b = -2$, and $c = 5$.

$b^2 - 4ac$

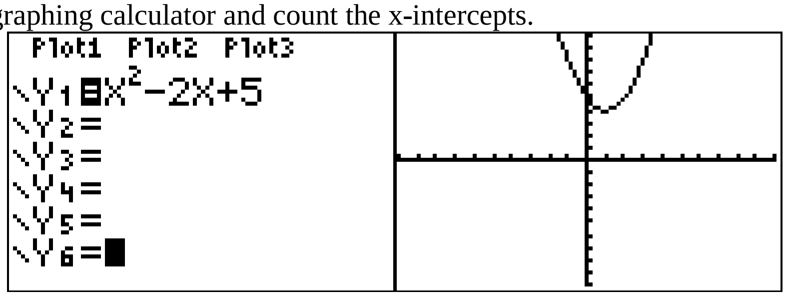
$(-2)^2 - 4(1)(5)$

$4 - 20$

$-16$

Because the value of the discriminant is negative, there are no real solutions.

Strategy 2.
Input the equation in a graphing calculator and count the x-intercepts.




The graph does not intercept the x-axis, so there are no real solutions.

Strategy 3
Solve the quadratic to see how many real solutions there are.

$$x^2 - 2x + 5 = 0$$

$$x^2 - 2x = -5$$

$$(x-1)^2 = -5 + (-1)^2$$

$$(x-1)^2 = -5 + 1$$

$$(x-1)^2 = -4$$

$$x - 1 = \sqrt{-4}$$

$$x - 1 = \pm 2i$$

$$x = 1 \pm 2i$$

Both solutions involve imaginary numbers, so there are no real solutions.

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207) ANS: 3

Strategy: Use the discriminant, which is $b^2 - 4ac$.

If the discriminant is > 0, then the quadratic has two real-number solutions.
If the discriminant is = 0, then the quadratic has one real-numer solution.
If the discriminant is < 0, the the quadratic has zero real-number solutions.

STEP 1. Identify the values of *a*, *b*, and *c* in the quadratic equation $4x^2 + 2x + 5 = 0$.

$$a = 4$$

$$b = 2$$

$$c = 5$$

STEP 2. Substitute the values into $b^2 - 4ac$ and evaluate.

$$b^2 - 4ac$$

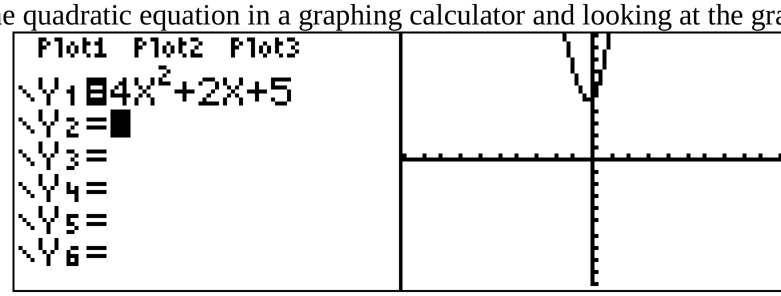
$$(2)^2 - 4(4)(5)$$

$$4 - 80$$

$$-76$$

The quadratic has zero real-number solutions.

CHECK by inputting the quadratic equation in a graphing calculator and looking at the graph view.




The number of solutions is equal to the number of x-axis intercepts. In this case, the parabola opens upward and does not cross the x-axis, which means it has zero real-numer solutions.

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