

**A2.A.68: Trigonometric Equations 7: Solve trigonometric equations for all values of the variable from  $0^\circ$  to  $360^\circ$**

- 1 Find all values of  $x$  in the interval  $0^\circ < x < 360^\circ$  that satisfy the equation  $3 \cos x + \sin 2x = 0$ .
- 2 Find all values of  $\theta$  in the interval  $0^\circ \leq \theta < 360^\circ$  that satisfy the equation  $\cos \theta - \sin 2\theta = 0$ .
- 3 Find all values of  $\theta$  in the interval  $0^\circ \leq \theta < 360^\circ$  that satisfy the equation  $\sin 2\theta = \sin \theta$ .
- 4 Solve the equation  $\cos 2x = \cos x$  algebraically for all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$ .
- 5 Find all values of  $\theta$  in the interval  $0^\circ \leq \theta \leq 360^\circ$  which satisfy the equation  $\sin \theta - \cos 2\theta = 0$ .
- 6 Find all values of  $\theta$  in the interval  $0^\circ \leq \theta < 360^\circ$  that satisfy the equation  $\cos 2\theta + 2 = \sin \theta$ .
- 7 In the interval  $0^\circ \leq A \leq 360^\circ$ , solve for all values of  $A$  in the equation  $\cos 2A = -3 \sin A - 1$ .
- 8 A solution of the equation  $\cos 2\theta + \sin 2\theta = -1$  is
  - 1)  $240^\circ$
  - 2)  $135^\circ$
  - 3)  $45^\circ$
  - 4)  $-30^\circ$
- 9 Find, to the *nearest ten minutes* or *nearest tenth of a degree*, all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$  that satisfy the equation  $2 \sin 2x + \cos x = 0$ .
- 10 Find, to the *nearest degree*, all values of  $\theta$  in the interval  $0^\circ < \theta < 360^\circ$  that satisfy the equation  $3 \cos 2\theta + \sin \theta - 1 = 0$ .

- 11 Find all values of  $\theta$  in the interval  $0^\circ \leq \theta \leq 360^\circ$  that satisfy the equation  $3 \cos 2\theta + 2 \sin \theta + 1 = 0$ , and round all answers to the *nearest hundredth of a degree*. [Only an algebraic solution can receive full credit.]
- 12 Find, to the *nearest degree*, all values of  $\theta$  in the interval  $0^\circ \leq \theta \leq 360^\circ$  which satisfy the equation  $3 \cos 2\theta + \sin \theta - 2 = 0$ .
- 13 Find all values of  $\theta$  in the interval  $0^\circ \leq \theta < 360^\circ$  that satisfy the equation  $5 \sin \theta + 2 \cos 2\theta - 3 = 0$ . Express your answer to the *nearest ten minutes* or *nearest tenth of a degree*.
- 14 Find, to the *nearest degree*, all values of  $x$  between  $0^\circ$  and  $360^\circ$  that satisfy the equation  $2 \sin x + 4 \cos 2x = 3$ .
- 15 Find all positive values of  $\theta$  less than  $360^\circ$  that satisfy the equation  $2 \cos 2\theta - 3 \sin \theta = 1$ . Express your answers to the *nearest ten minutes* or *nearest tenth of a degree*.
- 16 Find all values of  $\theta$  in the interval  $0^\circ \leq \theta \leq 360^\circ$  that satisfy the equation  $\sin \theta = 2 + 3 \cos 2\theta$ . Express your answer to the *nearest ten minutes* or *nearest tenth of a degree*.
- 17 Solve the equation  $\cos \theta = 2 + 3 \cos 2\theta$  for all values of  $\theta$ , to the *nearest tenth of a degree*, in the interval  $0^\circ \leq \theta < 360^\circ$ .
- 18 Find, to the *nearest degree*, all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$  that satisfy the equation  $3 \cos 2x + \cos x + 2 = 0$ .
- 19 Find, to the *nearest ten minutes* or *nearest tenth of a degree*, all values of  $x$  in the interval  $0^\circ \leq x < 360^\circ$  that satisfy the equation  $4 \cos 2x - 2 \cos x + 3 = 0$ .
- 20 Find all values of  $\theta$  in the interval  $0^\circ \leq \theta < 360^\circ$  that satisfy the equation  $3 \cos 2\theta = 7 \cos \theta$ . Express your answer to the *nearest tenth of a degree* or *nearest ten minutes*.

# A2.A.68: Trigonometric Equations 7: Solve trigonometric equations for all values of the variable from $0^\circ$ to $360^\circ$

## Answer Section

1 ANS:

$$\begin{aligned}
 3\cos x + \sin 2x &= 0 \\
 3\cos x + 2\sin x \cos x &= 0 \\
 \cos x(3 + 2\sin x) &= 0 \\
 90^\circ, 270^\circ. \quad \cos x = 0 \quad \text{or} \quad 3 + 2\sin x = 0 \\
 x &= \cos^{-1} 0 \quad \text{or} \quad \sin x = -\frac{3}{2} \\
 x &= 90^\circ, 270^\circ
 \end{aligned}$$

PTS: 4 REF: 010829b

2 ANS:

$30^\circ, 90^\circ, 150^\circ, 270^\circ$

PTS: 6 REF: 089341siii

3 ANS:

$$\begin{aligned}
 0, 60, 180, 300. \quad \sin 2\theta &= \sin \theta \\
 \sin 2\theta - \sin \theta &= 0 \\
 2\sin \theta \cos \theta - \sin \theta &= 0 \\
 \sin \theta(2\cos \theta - 1) &= 0 \\
 \sin \theta = 0 \quad 2\cos \theta - 1 &= 0 \\
 \theta = 0, 180 \quad \cos \theta &= \frac{1}{2} \\
 \theta &= 60, 300
 \end{aligned}$$

PTS: 2 REF: 061037a2

4 ANS:

$$\begin{aligned}
 2\cos^2 x - 1 &= \cos x \\
 2\cos^2 x - \cos x - 1 &= 0 \\
 (2\cos x + 1)(\cos x - 1) &= 0 \\
 \cos x &= -\frac{1}{2}, 1 \\
 x &= 0, 120, 240
 \end{aligned}$$

PTS: 4 REF: 011638a2

5 ANS:

$30^\circ, 150^\circ, 270^\circ$

PTS: 6 REF: 068541siii

6 ANS:  
90°

PTS: 5

REF: 088737siii

7 ANS:

$$\begin{aligned}
 & -2x^2 + 3x + 2 = 0 \\
 & 2x^2 - 3x - 2 = 0 \\
 & 210^\circ, 330^\circ. \quad 1 - 2\sin^2 A = -3\sin A - 1 \quad (2x+1)(x-2) = 0 \\
 & -2\sin^2 A + 3\sin A + 2 = 0 \quad 2x+1=0 \quad x-2=0 \\
 & \quad \quad \quad x = -\frac{1}{2} \quad x = 2
 \end{aligned}$$

$$\sin A = -\frac{1}{2} \quad \sin A = 2$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = -30^\circ, \text{ or } 330^\circ, \text{ and } 210^\circ.$$

$$\sin^{-1}(2) \text{ has no solution}$$

PTS: 4

REF: 060131b

8 ANS: 2

PTS: 2

REF: 060024siii

9 ANS:

$$90^\circ, 194.5^\circ, 270^\circ, 345.5^\circ \text{ or } 90^\circ, 194^\circ 30', 270^\circ, 345^\circ 30'$$

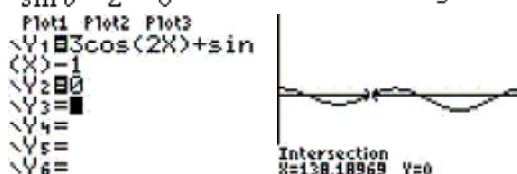
PTS: 10

REF: 069738siii

10 ANS:

$$\begin{aligned}
 & 3\cos 2\theta + \sin \theta - 1 = 0 \quad 6x^2 - x - 2 = 0 \\
 & 3(1 - 2\sin^2 \theta) + \sin \theta - 1 = 0 \quad (3x - 2)(2x + 1) = 0 \\
 & 42, 138, 210, 330. \quad 3 - 6\sin^2 \theta + \sin \theta - 1 = 0. \quad 3x - 2 = 0 \quad 2x + 1 = 0. \\
 & -6\sin^2 \theta + \sin \theta + 2 = 0 \quad x = \frac{2}{3} \quad x = -\frac{1}{2} \\
 & 6\sin^2 \theta - \sin \theta - 2 = 0
 \end{aligned}$$

$$\begin{aligned}
 & \sin \theta = \frac{2}{3} \quad \sin \theta = -\frac{1}{2} \\
 & \theta = \sin^{-1}\left(\frac{2}{3}\right) \quad \theta = \sin^{-1}\left(-\frac{1}{2}\right) \\
 & \theta \approx 42^\circ, 138^\circ \quad \theta \approx 210^\circ, 330^\circ
 \end{aligned}$$



PTS: 4

REF: 060530b

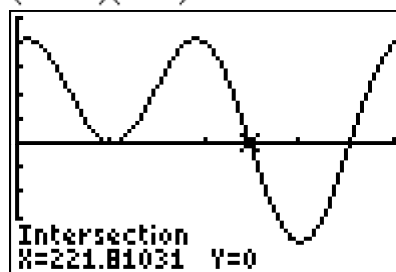
11 ANS:

$$\begin{aligned}
 3\cos 2\theta + 2\sin \theta + 1 &= 0 \\
 3(1 - 2\sin^2 \theta) + 2\sin \theta + 1 &= 0 & x = -\frac{2}{3} \text{ or } x = 1 \\
 3 - 6\sin^2 \theta + 2\sin \theta + 1 &= 0 \\
 -6\sin^2 \theta + 2\sin \theta + 4 &= 0 & \sin \theta = -\frac{2}{3} \text{ or } \sin \theta = 1 \\
 3\sin^2 \theta - \sin \theta - 2 &= 0 & \theta = \sin^{-1}\left(-\frac{2}{3}\right) \text{ or } \theta = \sin^{-1} 1 \\
 \text{let } \sin \theta = x & \\
 3x^2 - x - 2 &= 0 & \theta \approx 318.19 \text{ or } 221.81 \text{ or } \theta = 90 \\
 (3x + 2)(x - 1) &= 0
 \end{aligned}$$

```

Plot1 Plot2 Plot3
Y1=cos(2X)+2sin(X)+1
Y2=0
Y3=
Y4=
Y5=
Y6=

```



PTS: 4 REF: 060829b

12 ANS:

30°, 150°, 199°, 341°

PTS: 10 REF: 068137siii

13 ANS:

14°30', 90°, 165°30' or 14.5°, 90°, 165.5°

PTS: 10 REF: 010437siii

14 ANS:

30, 150, 194, 346

PTS: 10 REF: 089539siii

15 ANS:

14.5, 165.5, 270 or 14°30', 165°30', 270°

PTS: 10 REF: 019837siii

16 ANS:

56.4°, 123.6°, 270° or 56°30', 123°30', 270°

PTS: 10 REF: 060139siii

17 ANS:

$$\cos \theta = 2 + 3 \cos 2\theta$$

$$\cos \theta = 2 + 3(2 \cos^2 \theta - 1)$$

$$\cos \theta = 2 + 6 \cos^2 \theta - 3$$

$$0 = 6 \cos^2 \theta - \cos \theta - 1$$

$$60, 109.5, 250.5, 300. \quad 0 = 6x^2 - x - 1$$

$$0 = (3x + 1)(2x - 1)$$

$$x = -\frac{1}{3} \quad x = \frac{1}{2}$$

$$\cos \theta = -\frac{1}{3} \quad \cos \theta = \frac{1}{2}$$

$$\theta \approx 109.5^\circ, 250.5^\circ \quad \theta = 60^\circ, 300^\circ$$

PTS: 4

REF: 060932b

18 ANS:

71, 120, 240, 289

PTS: 10

REF: 069638siii

19 ANS:

60°, 104°30', 255°30' and 300° or 60°, 104.5°, 255.5° and 300°

PTS: 10

REF: 060337siii

20 ANS:

109°30', 250°30' or 109.5°, 250.5°

PTS: 10

REF: 080340siii