

A2.A.77: Double Angle Identities 4: Apply the double-angle and half-angle formulas for trigonometric functions

- 1 Is $\frac{1}{2} \sin 2x$ the same expression as $\sin x$? Justify your answer.
- 2 For all values of θ for which the expressions are defined, prove that the following is an identity:
$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$
- 3 For all values of θ for which the expressions are defined, prove the following is an identity:
$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$
- 4 For all value of x for which the expressions are defined, prove the following is an identity:
$$\sec x \sin 2x = \frac{2 - 2 \cos^2 x}{\sin x}$$
- 5 For all values of x for which the expressions are defined, show that the following equation is an identity: $\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$
- 6 For all values of θ for which the expressions are defined, prove that the following is an identity:
$$1 + \tan^2 \theta = \frac{2 \sin \theta}{\cos \theta \sin 2\theta}$$
- 7 Prove the identity: $\sin 2x = \tan x(2 - 2 \sin^2 x)$
- 8 For all values of A for which the expressions are defined, show that the following is an identity:
$$\frac{2 \sin^2 A}{\sin 2A} + \cot A = \sec A \csc A$$
- 9 For all values of θ for which the expressions are defined, prove the following is an identity:
$$\frac{\cos(90^\circ - \theta)}{\sin 2\theta} = \frac{\sec \theta}{2}$$
- 10 For all value of x for which the expressions are defined, prove the following is an identity:
$$2 \sin x \cos x = \tan 2x \cos 2x$$
- 11 For all values of A for which the expressions are defined, prove the following is an identity:
$$\cot A = \frac{1 + \cos A + \cos 2A}{\sin A + \sin 2A}$$

- 12 For all values of θ for which the expressions are defined, prove the following is an identity:

$$\frac{\sin 2\theta + \sin \theta}{\cos 2\theta + \cos \theta + 1} = \tan \theta$$

- 13 For all values of x for which the expressions are defined, prove that the following is an identity:
 $\tan x + \cot x = 2 \csc 2x$

- 14 For all value of x for which the expression is defined, prove that the following is an identity:

$$\cot x = \frac{\sin 2x}{1 - \cos 2x}$$

- 15 For all values of x for which the expressions are defined, prove the following is an identity:

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \csc x$$

- 16 Express $\frac{\cos 2A + \sin^2 A}{\cos A}$ as a single trigonometric function for all values of A for which the fraction is defined.

- 17 For all value of x for which the expressions are defined, prove the following is an identity:

$$\frac{\cos 2x}{\sin x} + \sin x = \csc x - \sin x$$

- 18 Prove the following identity:

$$\frac{\sin \theta}{\sin^2 \theta + \cos 2\theta} = \frac{\sec \theta}{\cot \theta}$$

- 19 Prove the following identity:

$$\frac{(\sin x + \cos x)^2 - 1}{\cos x} = (\sin 2x)(\tan x)(\csc x)$$

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Answer Section

1 ANS:

$$\frac{1}{2} \sin 2x = \sin x$$

$$\frac{1}{2} (2 \sin x \cos x) = \sin x$$

$$\sin x \cos x = \sin x$$

$$\sin x \cos x - \sin x = 0$$

Not an identity, so the expressions are not equal.

$$\sin x (\cos x - 1) = 0$$

$$\sin x = 0 \text{ or } \cos x - 1 = 0$$

$$x = 0^\circ \text{ or } 180^\circ$$

REF: 060222b

2 ANS:

$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$2 \sin \theta \cos \theta = \frac{\frac{2 \sin \theta}{\cos \theta}}{\frac{1}{\cos^2 \theta}}$$

$$\cos \theta = \frac{\frac{1}{\cos \theta}}{\frac{1}{\cos^2 \theta}}$$

$$\cos \theta = \cos \theta$$

REF: 088442siii

3 ANS:

$$\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$\cos 2\theta = \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{\sec^2 \theta}$$

$$\cos 2\theta = \frac{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}{\frac{1}{\cos^2 \theta}}$$

$$\cos^2 \theta - \sin^2 \theta = \left(1 - \frac{\sin^2 \theta}{\cos^2 \theta} \right) \cos^2 \theta$$

$$\cos^2 \theta - \sin^2 \theta = \cos^2 \theta - \sin^2 \theta$$

REF: 018542siii

4 ANS:

$$\sec x \sin 2x = \frac{2 - 2\cos^2 x}{\sin x}$$

$$\frac{1}{\cos x} \cdot 2\sin x \cos x = \frac{2(1 - \cos^2 x)}{\sin x}$$

$$\sin x = \frac{\sin^2 x}{\sin x}$$

$$\sin x = \sin x$$

REF: 068541siii

5 ANS:

$$\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$2 \sin x \cos x = \frac{\frac{2 \sin x}{\cos x}}{\sec^2 x}$$

$$2 \sin x \cos x = \frac{\frac{2 \sin x}{\cos x}}{\frac{1}{\cos^2 x}}$$

$$\frac{2 \sin x}{\cos x} = \frac{2 \sin x}{\cos x}$$

REF: 068738siii

6 ANS:

$$\sec^2 \theta = \frac{2 \sin \theta}{\cos \theta \cdot 2 \sin \theta \cdot \cos \theta}$$

$$\frac{1}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

REF: 088737siii

7 ANS:

$$\sin 2x = \tan x (2 - 2 \sin^2 x)$$

$$2 \sin x \cos x = \frac{\sin x}{\cos x} (2(1 - \sin^2 x))$$

$$2 \cos x = \frac{(2(\cos^2 x))}{\cos x}$$

$$\cos x = \cos x$$

REF: 018941siii

8 ANS:

$$\frac{2 \sin^2 A}{\sin 2A} + \cot A = \sec A \csc A$$

$$\frac{2 \sin^2 A}{2 \sin A \cos A} + \frac{\cos A}{\sin A} = \frac{1}{\cos A} \cdot \frac{1}{\sin A}$$

$$\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{1}{\cos A} \cdot \frac{1}{\sin A}$$

$$\frac{\sin^2 A + \cos^2 A}{\cos A \cdot \sin A} = \frac{1}{\cos A \cdot \sin A}$$

$$\frac{1}{\cos A \cdot \sin A} = \frac{1}{\cos A \cdot \sin A}$$

REF: 068942siii

9 ANS:

$$\frac{\cos(90 - \theta)}{\sin 2\theta} = \frac{\sec \theta}{2}$$

$$\frac{\cos 90 \cos \theta + \sin 90 \sin \theta}{2 \sin \theta \cos \theta} = \frac{\sec \theta}{2}$$

$$\frac{\sin \theta}{2 \sin \theta \cos \theta} = \frac{\sec \theta}{2}$$

$$\frac{\sec \theta}{2} = \frac{\sec \theta}{2}$$

REF: 069040siii

10 ANS:

$$2 \sin x \cos x = \tan 2x \cos 2x$$

$$\sin 2x = \frac{\sin 2x}{\cos 2x} \cos 2x$$

$$\sin 2x = \sin 2x$$

REF: 069442siii

11 ANS:

$$\cot A = \frac{1 + \cos A + \cos 2A}{\sin A + \sin 2A}$$

$$\frac{\cos A}{\sin A} = \frac{1 + \cos A + 2 \cos^2 A - 1}{\sin A + 2 \sin A \cos A}$$

$$\frac{\cos A}{\sin A} = \frac{\cos A(1 + 2 \cos A)}{\sin A(1 + 2 \cos A)}$$

$$\frac{\cos A}{\sin A} = \frac{\cos A}{\sin A}$$

REF: 089442siii

12 ANS:

$$\frac{\sin 2\theta + \sin \theta}{\cos 2\theta + \cos \theta + 1} = \tan \theta$$

$$\frac{2 \sin \theta \cos \theta + \sin \theta}{2 \cos^2 \theta - 1 + \cos \theta + 1} = \tan \theta$$

$$\frac{\sin \theta(2 \cos \theta + 1)}{\cos \theta(2 \cos \theta + 1)} = \tan \theta$$

$$\tan \theta = \tan \theta$$

REF: 019637siii

13 ANS:

$$\tan x + \cot x = 2 \csc 2x$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{2}{\sin 2x}$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{2}{2 \sin x \cos x}$$

$$\frac{1}{\sin x \cos x} = \frac{1}{\sin x \cos x}$$

REF: 069640siii

14 ANS:

$$\cot x = \frac{\sin 2x}{1 - \cos 2x}$$

$$\frac{\cos x}{\sin x} = \frac{2 \sin x \cos x}{1 - (1 - 2 \sin^2 x)}$$

$$\frac{\cos x}{\sin x} = \frac{2 \sin x \cos x}{2 \sin^2 x}$$

$$\frac{\cos x}{\sin x} = \frac{\cos x}{\sin x}$$

REF: 089638siii

15 ANS:

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \csc x$$

$$\frac{1 - 2 \sin^2 x}{\sin x} + \frac{2 \sin x \cos x}{\cos x} = \frac{1}{\sin x}$$

$$\frac{1 - 2 \sin^2 x}{\sin x} + 2 \sin x = \frac{1}{\sin x}$$

$$\frac{1 - 2 \sin^2 x + 2 \sin^2 x}{\sin x} = \frac{1}{\sin x}$$

$$\frac{1}{\sin x} = \frac{1}{\sin x}$$

REF: 019740siii

16 ANS:

 $\cos A$

REF: 010014siii

17 ANS:

$$\frac{\cos 2x}{\sin x} + \sin x = \csc x - \sin x$$

$$\frac{\cos 2x}{\sin x} + \sin x = \frac{1}{\sin x} - \sin x$$

$$\cos 2x + \sin^2 x = 1 - \sin^2 x$$

$$1 - 2 \sin^2 x + \sin^2 x = 1 - \sin^2 x$$

$$1 - \sin^2 x = 1 - \sin^2 x$$

REF: 060141siii

18 ANS:

$$\frac{\sin \theta}{\sin^2 \theta + \cos 2\theta} = \frac{\sec \theta}{\cot \theta}$$

$$\frac{\sin \theta}{\sin^2 \theta + \cos^2 \theta - \sin^2 \theta} = \frac{\frac{1}{\cos \theta}}{\frac{\cos \theta}{\sin \theta}}$$

$$\frac{\sin \theta}{\cos^2 \theta} = \frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta}$$

$$\frac{\sin \theta}{\cos^2 \theta} = \frac{\sin \theta}{\cos^2 \theta}$$

REF: 010242siii

19 ANS:

$$\frac{(\sin x + \cos x)^2 - 1}{\cos x} = (\sin 2x)(\tan x)(\csc x)$$

$$\frac{\sin^2 x + 2 \sin x \cos x + \cos^2 x - 1}{\cos x} = 2 \sin x \cos x \left(\frac{\sin x}{\cos x} \right) \left(\frac{1}{\sin x} \right)$$

$$\frac{\sin^2 x + 2 \sin x \cos x - \sin^2 x}{\cos x} = 2 \sin x$$

$$\frac{2 \sin x \cos x}{\cos x} = 2 \sin x$$

$$2 \sin x = 2 \sin x$$

REF: 080239siii