

### A2.A.67: Proving Trigonometric Identities: Justify the Pythagorean identities

- 1 Starting with  $\sin^2 A + \cos^2 A = 1$ , derive the formula  $\tan^2 A + 1 = \sec^2 A$ .
- 2 Show that  $\sec \theta \sin \theta \cot \theta = 1$  is an identity.
- 3 A crate weighing  $w$  pounds sits on a ramp positioned at an angle of  $\theta$  with the horizontal. The forces acting on this crate are modeled by the equation  $Mw \cos \theta = w \sin \theta$ , where  $M$  is the coefficient of friction. What is an expression for  $M$  in terms of  $\theta$ ?
  - 1)  $M = \tan \theta$
  - 2)  $M = \cot \theta$
  - 3)  $M = \sec \theta$
  - 4)  $M = \csc \theta$
- 4 For all values of  $\theta$  for which the expressions are defined, prove the identity:  
 $\tan \theta + \cot \theta = \sec \theta \csc \theta$
- 5 For all values of  $\theta$  for which the expressions are defined, prove the identity:  
 $\frac{\tan \theta \csc^2 \theta}{1 + \tan^2 \theta} = \cot \theta$
- 6 For all values of  $\theta$  for which the expressions are defined, prove that the following is an identity:  
 $\frac{(\cos \theta + \sin \theta)^2}{1 + 2 \sin \theta \cos \theta} = \cos \theta \tan \theta \csc \theta$
- 7 For all values of  $\theta$  for which the expressions are defined, prove the following is an identity:  
 $\frac{\cos \theta \sin \theta + \cos \theta}{\cos^2 \theta} = \tan \theta + \sec \theta$
- 8 For all values of  $\theta$  for which the expressions are defined, prove that the following is an identity:  
 $\cos \theta (\cos \theta + 1) + \sin^2 \theta = \frac{\sin \theta + \tan \theta}{\tan \theta}$
- 9 For all values of  $\theta$  for which the expressions are defined, prove that the following is an identity:  
 $\sec \theta - \sin \theta \tan \theta = \cos \theta$
- 10 For all values of  $x$  for which the expressions are defined, prove that the following is an identity:  
 $\frac{\tan x \csc^2 x}{1 + \tan^2 x} = \cot x$
- 11 For all values of  $x$  for which the expressions are defined, prove the following equation is an identity:  
 $\frac{2}{\sec x} = \frac{2 - 2 \sin^2 x}{\cos x}$
- 12 For all values of  $x$  for which the expressions are defined, prove that the following is an identity:  
 $\frac{\cos x + \cot x}{1 + \csc x} = \cos x$

- 13 For all values of  $x$  for which the expressions are defined, prove the following is an identity:  
 $\sec^2 x + \csc^2 x = (\tan x + \cot x)^2$

- 20 Prove the following identity:  
 $\tan x + \cot x = \csc x \sec x$

- 14 For all values of  $x$  for which the expressions are defined, prove that the following is an identity:  
 $\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$

- 21 Prove that the equation shown below is an identity for all values for which the functions are defined:  
 $\csc \theta \cdot \sin^2 \theta \cdot \cot \theta = \cos \theta$

- 15 Prove the following identity:  
 $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \cot \theta \sec \theta$

- 16 Prove the following is an identity for all values of  $\theta$  for which the expressions are defined:  
 $\frac{\sin \theta}{\cot \theta} + \cos \theta = \sec \theta$

- 17 For all values of  $\theta$  for which the expressions are defined, prove the following is an identity:  
 $\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = 2 \sin^2 \theta - 1$

- 18 For all values of  $\theta$  for which the expressions are defined, prove the following is an identity:  
 $(\cot \theta + \csc \theta)(1 - \cos \theta) = \sin \theta$

- 19 Prove the following identity:  $\frac{\tan \theta}{\cot \theta} + 1 = \sec^2 \theta$

## A2.A.67: Proving Trigonometric Identities: Justify the Pythagorean identities

### Answer Section

1 ANS:

$$\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$$

$$\tan^2 A + 1 = \sec^2 A$$

REF: 011135a2

2 ANS:

$$\sec \theta \sin \theta \cot \theta = \frac{1}{\cos \theta} \cdot \sin \theta \cdot \frac{\cos \theta}{\sin \theta} = 1$$

REF: 011428a2

3 ANS: 1

$$M \cos \theta = \sin \theta$$

$$M \cos \theta = \sin \theta$$

$$M = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

REF: 060515b

4 ANS:

$$\tan \theta + \cot \theta = \sec \theta \csc \theta$$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta \cdot \sin \theta} = \frac{1}{\cos \theta \cdot \sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

REF: 068036siii

5 ANS:

$$\frac{\tan \theta \csc^2 \theta}{1 + \tan^2 \theta} = \cot \theta$$

$$\frac{\frac{\sin \theta}{\cos \theta} \frac{1}{\sin^2 \theta}}{\sec^2 \theta} = \cot \theta$$

$$\frac{\frac{\sin \theta}{\cos \theta} \frac{1}{\sin^2 \theta}}{\frac{1}{\cos^2 \theta}} = \cot \theta$$

$$\frac{1}{\cos \theta \sin \theta} \cos^2 \theta = \frac{\cos \theta}{\sin \theta}$$

$$\frac{\cos \theta}{\sin \theta} = \frac{\cos \theta}{\sin \theta}$$

REF: 068140siii

6 ANS:

$$\frac{(\cos \theta + \sin \theta)^2}{1 + 2 \sin \theta \cos \theta} = \cos \theta \tan \theta \csc \theta$$

$$\frac{\cos^2 \theta + 2 \cos \theta \sin \theta + \sin^2 \theta}{1 + 2 \sin \theta \cos \theta} = \cos \theta \frac{\sin \theta}{\cos \theta} \frac{1}{\sin \theta}$$

$$\frac{1 + 2 \cos \theta \sin \theta}{1 + 2 \sin \theta \cos \theta} = 1$$

REF: 068440siii

7 ANS:

$$\frac{\cos \theta \sin \theta + \cos \theta}{\cos^2 \theta} = \tan \theta + \sec \theta$$

$$\frac{\cos \theta (\sin \theta + 1)}{\cos^2 \theta} = \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta}$$

$$\frac{\sin \theta + 1}{\cos \theta} = \frac{\sin \theta + 1}{\cos \theta}$$

REF: 068639siii

8 ANS:

$$\cos \theta (\cos \theta + 1) + \sin^2 \theta = \frac{\sin \theta + \tan \theta}{\tan \theta}$$

$$\cos^2 \theta + \cos \theta + \sin^2 \theta = \frac{\sin \theta}{\tan \theta} + 1$$

$$\cos \theta + 1 = \frac{\sin \theta}{\frac{\sin \theta}{\cos \theta}} + 1$$

$$\cos \theta + 1 = \cos \theta + 1$$

REF: 088636siii

9 ANS:

$$\sec \theta - \sin \theta \tan \theta = \cos \theta$$

$$\frac{1}{\cos \theta} - \sin \theta \frac{\sin \theta}{\cos \theta} = \cos \theta$$

$$1 - \sin^2 \theta = \cos^2 \theta$$

$$\cos^2 \theta = \cos^2 \theta$$

REF: 088937siii

10 ANS:

$$\frac{\tan x \csc^2 x}{1 + \tan^2 x} = \cot x$$

$$\frac{\frac{\sin x}{\cos x} \cdot \frac{1}{\sin^2 x}}{\sec^2 x} = \cot x$$

$$\frac{\frac{1}{\cos x \cdot \sin x}}{\frac{1}{\cos^2 x}} = \cot x$$

$$\frac{\cos^2 x}{\cos x \cdot \sin x} = \cot x$$

$$\frac{\cos x}{\sin x} = \frac{\cos x}{\sin x}$$

REF: 089038siii

11 ANS:

$$\frac{2}{\sec x} = \frac{2 - 2\sin^2 x}{\cos x}$$

$$\frac{2}{\frac{1}{\cos x}} = \frac{2(1 - \sin^2 x)}{\cos x}$$

$$\cos x = \frac{\cos^2 x}{\cos x}$$

$$\cos x = \cos x$$

REF: 089341siii

12 ANS:

$$\frac{\cos x + \cot x}{1 + \csc x} = \cos x$$

$$\frac{\cos x + \frac{\cos x}{\sin x}}{1 + \frac{1}{\sin x}} = \cos x$$

$$\cos x + \frac{\cos x}{\sin x} = \cos x + \frac{\cos x}{\sin x}$$

REF: 019439siii

13 ANS:

$$\sec^2 x + \csc^2 x = (\tan x + \cot x)^2$$

$$\tan^2 x + 1 + 1 + \cot^2 x = \tan^2 x + 2 \tan x \cot x + \cot^2 x$$

$$2 = 2 \frac{\sin x}{\cos x} \frac{\cos x}{\sin x}$$

$$2 = 2$$

REF: 019542siii

14 ANS:

$$\frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$$

$$\frac{\frac{1}{\cos x} + \frac{1}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \sin x + \cos x$$

$$\frac{\frac{\sin x + \cos x}{\sin x \cos x}}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}} = \sin x + \cos x$$

$$\frac{\sin x + \cos x}{\sin x \cos x} \cdot \frac{\sin x \cos x}{1} = \sin x + \cos x$$

$$\sin x + \cos x = \sin x + \cos x$$

REF: 069541siii

15 ANS:

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \cot \theta \sec \theta$$

$$\frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)} = \frac{2 \cos \theta}{\sin \theta} \frac{1}{\cos \theta}$$

$$\frac{\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta}{\sin \theta (1 + \cos \theta)} = \frac{2}{\sin \theta}$$

$$\frac{2 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)} = \frac{2}{\sin \theta}$$

$$\frac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)} = \frac{2}{\sin \theta}$$

$$\frac{2}{\sin \theta} = \frac{2}{\sin \theta}$$

REF: 019938siii

16 ANS:

$$\frac{\sin \theta}{\cot \theta} + \cos \theta = \sec \theta$$

$$\frac{\sin \theta + \cot \theta \cos \theta}{\cot \theta} = \frac{1}{\cos \theta}$$

$$\cos \theta (\sin \theta + \cot \theta \cos \theta) = \cot \theta$$

$$\cos \theta (\sin \theta + \cot \theta \cos \theta) = \frac{\cos \theta}{\sin \theta}$$

$$\sin \theta + \cot \theta \cos \theta = \frac{1}{\sin \theta}$$

$$\sin \theta (\sin \theta + \cot \theta \cos \theta) = 1$$

$$\sin \theta (\sin \theta + \frac{\cos \theta}{\sin \theta} \cos \theta) = 1$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

REF: 089940siii

17 ANS:

$$\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = 2 \sin^2 \theta - 1$$

$$\frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} = -(1 - 2 \sin^2 \theta)$$

$$\frac{\frac{\sin \theta}{\cos \theta} - \frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}} \times \frac{\cos \theta \sin \theta}{\cos \theta \sin \theta} = -(\cos 2\theta)$$

$$\frac{\sin^2 \theta - \cos^2 \theta}{\sin^2 \theta + \cos^2 \theta} = -(\cos 2\theta)$$

$$\sin^2 \theta - \cos^2 \theta = -(\cos 2\theta)$$

$$-(\cos 2\theta) = -(\cos 2\theta)$$

REF: 010042siii



18 ANS:

$$(\cot \theta + \csc \theta)(1 - \cos \theta) = \sin \theta$$

$$\left( \frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta} \right) (1 - \cos \theta) = \sin \theta$$

$$\sin \theta \cdot \left( \frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta} \right) (1 - \cos \theta) = \sin \theta \cdot \sin \theta$$

$$(\cos \theta + 1)(1 - \cos \theta) = \sin^2 \theta$$

$$(1 - \cos^2 \theta) = \sin^2 \theta$$

$$\sin^2 \theta = \sin^2 \theta$$

REF: 060041siii

19 ANS:

$$\frac{\tan \theta}{\cot \theta} + 1 = \sec^2 \theta$$

$$\frac{\frac{\sin \theta}{\cos \theta}}{\frac{\cos \theta}{\sin \theta}} + 1 = \frac{1}{\cos^2 \theta}$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} + 1 = \frac{1}{\cos^2 \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

REF: 080040siii

20 ANS:

$$\tan x + \cot x = \csc x \sec x$$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{1}{\sin x} \cdot \frac{1}{\cos x}$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x} = \frac{1}{\cos x \cdot \sin x}$$

$$\frac{1}{\cos x \cdot \sin x} = \frac{1}{\cos x \cdot \sin x}$$

REF: 060339siii

21 ANS:

$$\frac{1}{\sin \theta} \cdot \sin^2 \theta \cdot \frac{\cos \theta}{\sin \theta} = \cos \theta$$

$$\cos \theta = \cos \theta$$

REF: 011634a2