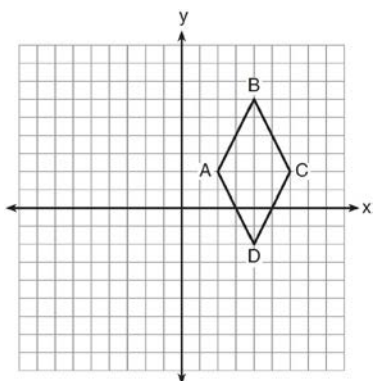


**G.G.69: Quadrilaterals in the Coordinate Plane 1: Investigate the properties of quadrilaterals in the coordinate plane, using the distance, midpoint, and slope formulas**

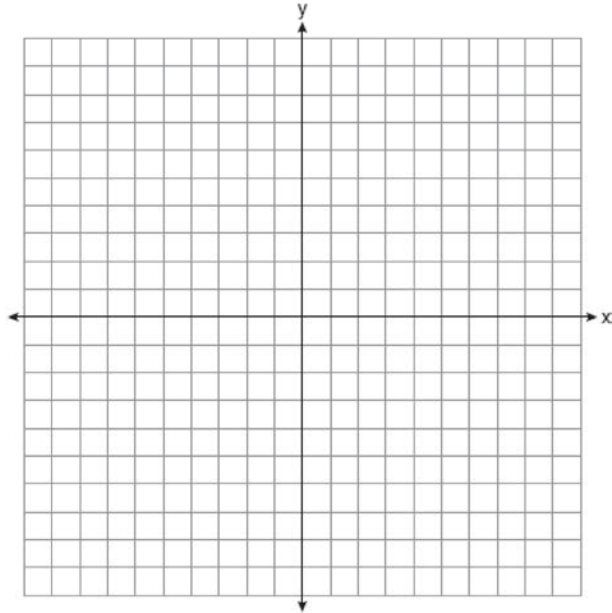
- 1 Quadrilateral  $ABCD$  is graphed on the set of axes below.



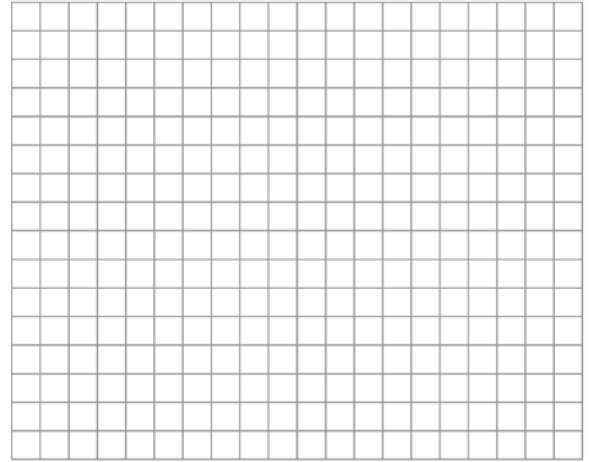
Which quadrilateral best classifies  $ABCD$ ?

- 1) trapezoid
  - 2) rectangle
  - 3) rhombus
  - 4) square
- 2 The coordinates of the vertices of parallelogram  $ABCD$  are  $A(-3, 2)$ ,  $B(-2, -1)$ ,  $C(4, 1)$ , and  $D(3, 4)$ . The slopes of which line segments could be calculated to show that  $ABCD$  is a rectangle?
- 1)  $\overline{AB}$  and  $\overline{DC}$
  - 2)  $\overline{AB}$  and  $\overline{BC}$
  - 3)  $\overline{AD}$  and  $\overline{BC}$
  - 4)  $\overline{AC}$  and  $\overline{BD}$
- 3 Square  $ABCD$  has vertices  $A(-2, -3)$ ,  $B(4, -1)$ ,  $C(2, 5)$ , and  $D(-4, 3)$ . What is the length of a side of the square?
- 1)  $2\sqrt{5}$
  - 2)  $2\sqrt{10}$
  - 3)  $4\sqrt{5}$
  - 4)  $10\sqrt{2}$
- 4 Parallelogram  $ABCD$  has coordinates  $A(1, 5)$ ,  $B(6, 3)$ ,  $C(3, -1)$ , and  $D(-2, 1)$ . What are the coordinates of  $E$ , the intersection of diagonals  $\overline{AC}$  and  $\overline{BD}$ ?
- 1)  $(2, 2)$
  - 2)  $(4.5, 1)$
  - 3)  $(3.5, 2)$
  - 4)  $(-1, 3)$
- 5 Rectangle  $KLMN$  has vertices  $K(0, 4)$ ,  $L(4, 2)$ ,  $M(1, -4)$ , and  $N(-3, -2)$ . Determine and state the coordinates of the point of intersection of the diagonals.
- 6 The coordinates of two vertices of square  $ABCD$  are  $A(2, 1)$  and  $B(4, 4)$ . Determine the slope of side  $\overline{BC}$ .

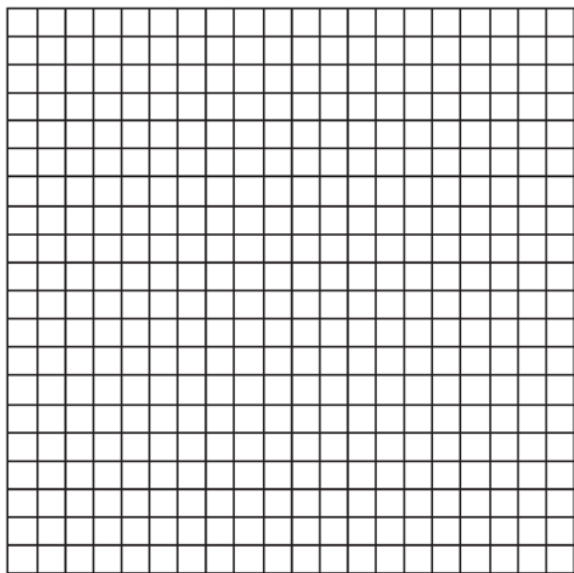
- 7 The vertices of quadrilateral  $JKLM$  have coordinates  $J(-3, 1)$ ,  $K(1, -5)$ ,  $L(7, -2)$ , and  $M(3, 4)$ . Prove that  $JKLM$  is a parallelogram. Prove that  $JKLM$  is *not* a rhombus. [The use of the set of axes below is optional.]



- 8 Given: Quadrilateral  $ABCD$  has vertices  $A(-5, 6)$ ,  $B(6, 6)$ ,  $C(8, -3)$ , and  $D(-3, -3)$ . Prove: Quadrilateral  $ABCD$  is a parallelogram but is neither a rhombus nor a rectangle. [The use of the grid below is optional.]



- 9 Quadrilateral  $MATH$  has coordinates  $M(1,1)$ ,  $A(-2,5)$ ,  $T(3,5)$ , and  $H(6,1)$ . Prove that quadrilateral  $MATH$  is a rhombus and prove that it is *not* a square. [The use of the grid is optional.]

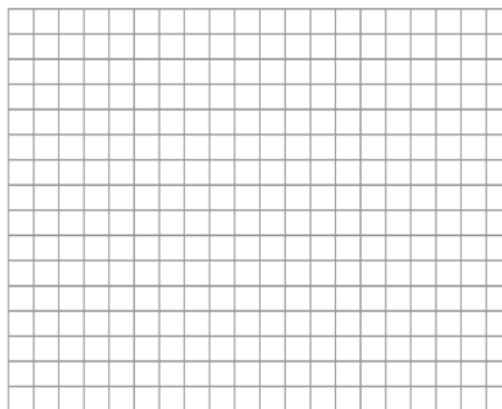


- 10 Given:  $\triangle ABC$  with vertices  $A(-6,-2)$ ,  $B(2,8)$ , and  $C(6,-2)$ .  $\overline{AB}$  has midpoint  $D$ ,  $\overline{BC}$  has midpoint  $E$ , and  $\overline{AC}$  has midpoint  $F$ .

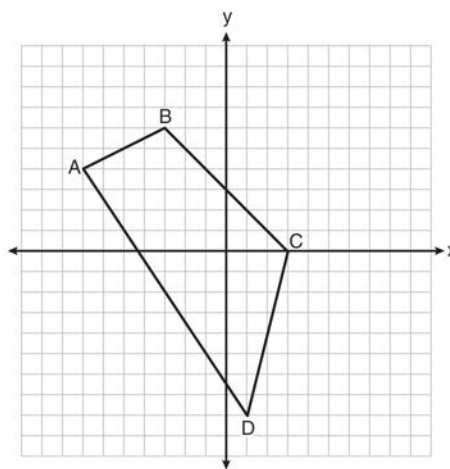
Prove:  $ADEF$  is a parallelogram

$ADEF$  is *not* a rhombus

[The use of the grid is optional.]



- 11 Quadrilateral  $ABCD$  with vertices  $A(-7,4)$ ,  $B(-3,6)$ ,  $C(3,0)$ , and  $D(1,-8)$  is graphed on the set of axes below. Quadrilateral  $MNPQ$  is formed by joining  $M$ ,  $N$ ,  $P$ , and  $Q$ , the midpoints of  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$ , and  $\overline{AD}$ , respectively. Prove that quadrilateral  $MNPQ$  is a parallelogram. Prove that quadrilateral  $MNPQ$  is *not* a rhombus.



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**Answer Section**

1 ANS: 3

Both pairs of opposite sides are parallel, so not a trapezoid. None of the angles are right angles, so not a rectangle or square. All sides are congruent, so a rhombus.

REF: 081411ge

2 ANS: 2

Adjacent sides of a rectangle are perpendicular and have opposite and reciprocal slopes.

REF: 061028ge

3 ANS: 2

$$\sqrt{(-2-4)^2 + (-3-(-1))^2} = \sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$$

REF: 011313ge

4 ANS: 1

The diagonals of a parallelogram intersect at their midpoints.  $M_{AC}\left(\frac{1+3}{2}, \frac{5+(-1)}{2}\right) = (2,2)$

REF: 061209ge

5 ANS:

$$\left(\frac{0+1}{2}, \frac{4+(-4)}{2}\right)$$

$$\left(\frac{1}{2}, 0\right)$$

REF: 081534ge

6 ANS:

$$m_{AB} = \frac{4-1}{4-2} = \frac{3}{2}. \quad m_{BC} = -\frac{2}{3}$$

REF: 061334ge

7 ANS:

$$m_{\overline{JM}} = \frac{1-4}{-3-3} = \frac{-3}{-6} = \frac{1}{2} \quad \text{Since both opposite sides have equal slopes and are parallel, } JKLM \text{ is a parallelogram.}$$

$$m_{\overline{ML}} = \frac{4-2}{3-7} = \frac{6}{-4} = -\frac{3}{2}$$

$$m_{\overline{LK}} = \frac{-2-5}{7-1} = \frac{3}{6} = \frac{1}{2}$$

$$m_{\overline{KJ}} = \frac{-5-1}{1-3} = \frac{-6}{4} = -\frac{3}{2}$$

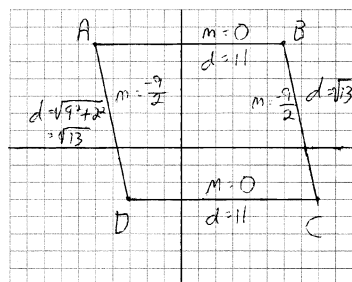
$$\overline{JM} = \sqrt{(-3-3)^2 + (1-4)^2} = \sqrt{45}. \quad \overline{JM} \text{ is not congruent to } \overline{ML}, \text{ so } JKLM \text{ is not a rhombus since not all sides}$$

$$\overline{ML} = \sqrt{(7-3)^2 + (-2-4)^2} = \sqrt{52}$$

are congruent.

REF: 061438ge

8 ANS:



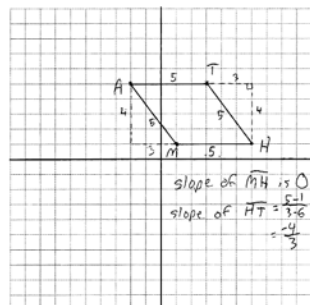
$\overline{AB} \parallel \overline{CD}$  and  $\overline{AD} \parallel \overline{CB}$  because their slopes are equal.  $ABCD$  is a parallelogram

because opposite sides are parallel.  $\overline{AB} \neq \overline{BC}$ .  $ABCD$  is not a rhombus because all sides are not equal.

$\overline{AB} \not\sim \perp \overline{BC}$  because their slopes are not opposite reciprocals.  $ABCD$  is not a rectangle because  $\angle ABC$  is not a right angle.

REF: 081038ge

9 ANS:



The length of each side of quadrilateral is 5. Since each side is congruent, quadrilateral

$MATH$  is a rhombus. The slope of  $\overline{MH}$  is 0 and the slope of  $\overline{HT}$  is  $-\frac{4}{3}$ . Since the slopes are not negative reciprocals, the sides are not perpendicular and do not form right angles. Since adjacent sides are not perpendicular, quadrilateral  $MATH$  is not a square.

REF: 011138ge

10 ANS:

$m_{\overline{AB}} = \left( \frac{-6+2}{2}, \frac{-2+8}{2} \right) = D(2,3)$   $m_{\overline{BC}} = \left( \frac{2+6}{2}, \frac{8+-2}{2} \right) = E(4,3)$   $F(0,-2)$ . To prove that  $ADEF$  is a parallelogram, show that both pairs of opposite sides of the parallelogram are parallel by showing the opposite sides have the same slope:  $m_{\overline{AD}} = \frac{3-2}{-2-6} = \frac{5}{4}$   $\overline{AF} \parallel \overline{DE}$  because all horizontal lines have the same slope.  $ADEF$

$$m_{\overline{FE}} = \frac{3-2}{4-0} = \frac{5}{4}$$

is not a rhombus because not all sides are congruent.  $AD = \sqrt{5^2 + 4^2} = \sqrt{41}$   $AF = 6$

REF: 081138ge

11 ANS:

$M\left(\frac{-7+-3}{2}, \frac{4+6}{2}\right) = M(-5,5)$   $m_{\overline{MN}} = \frac{5-3}{-5-0} = \frac{2}{-5}$ . Since both opposite sides have equal slopes and are

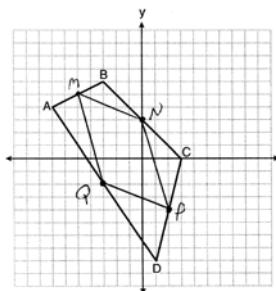
$$N\left(\frac{-3+3}{2}, \frac{6+0}{2}\right) = N(0,3) \quad m_{\overline{PQ}} = \frac{-4-2}{2-3} = \frac{-2}{5}$$

$$P\left(\frac{3+1}{2}, \frac{0+-8}{2}\right) = P(2,-4) \quad m_{\overline{NA}} = \frac{3-4}{0-2} = \frac{7}{-2}$$

$$Q\left(\frac{-7+1}{2}, \frac{4+-8}{2}\right) = Q(-3,-2) \quad m_{\overline{QM}} = \frac{-2-5}{-3--5} = \frac{-7}{2}$$

parallel,  $MNPQ$  is a parallelogram.  $\overline{MN} = \sqrt{(-5-0)^2 + (5-3)^2} = \sqrt{29}$ .  $\overline{MN}$  is not congruent to  $\overline{NP}$ , so  $MNPQ$

$$\overline{NA} = \sqrt{(0-2)^2 + (3-4)^2} = \sqrt{5}$$



is not a rhombus since not all sides are congruent.

REF: 081338ge