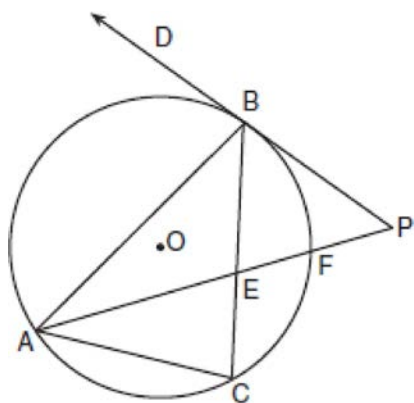


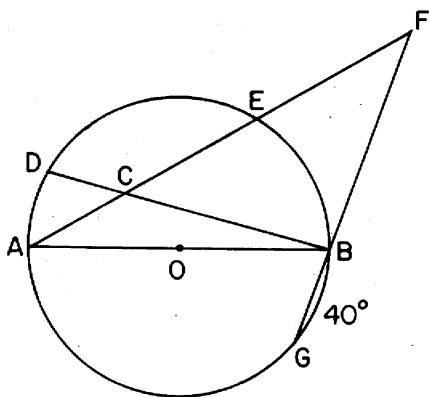
**G.G.51: Arcs Determined by Angles 7: Investigate theorems about the arcs determined by angles intersecting a circle when the vertex is on the circle**

- 1 In the accompanying diagram,  $\triangle ABC$  is inscribed in circle  $O$ ,  $\overrightarrow{AP}$  bisects  $\angle BAC$ ,  $\overrightarrow{PBD}$  is tangent to circle  $O$  at  $B$ , and  $m\angle ACB : m\angle CAB : m\angle ABC = 4 : 3 : 2$



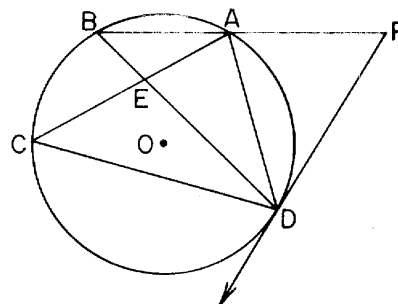
Find:  $m\angle ABC$ ,  $m\widehat{BF}$ ,  $m\angle BEP$ ,  $m\angle P$ ,  $m\angle PBC$

- 2 In the accompanying diagram,  $\overline{AB}$  is a diameter of circle  $O$ ,  $\overline{FECA}$  and  $\overline{FBG}$  are secants,  $m\widehat{AD} : m\widehat{DE} : m\widehat{EB} = 1 : 3 : 2$ .



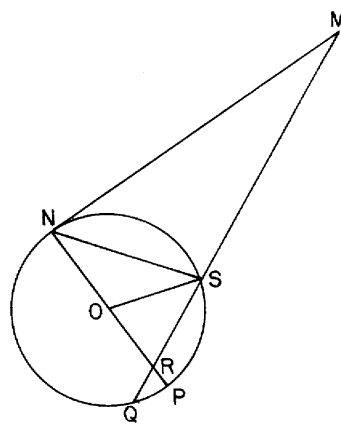
Find  $m\widehat{DE}$ ,  $m\angle ECB$ ,  $m\angle AFG$ ,  $m\angle DBF$ , and  $m\angle EAB$

- 3 In the accompanying diagram,  $B$  is the midpoint of  $\widehat{AC}$ , triangle  $ADC$  is inscribed in circle  $O$ , chords  $\overline{AC}$  and  $\overline{BD}$  intersect at  $E$ ,  $\overrightarrow{PR}$  is a tangent to circle  $O$  at  $D$ ,  $\overline{PAB}$  is a secant, and  $m\widehat{BA} : m\widehat{AD} : m\widehat{DC} = 2 : 3 : 5$ .



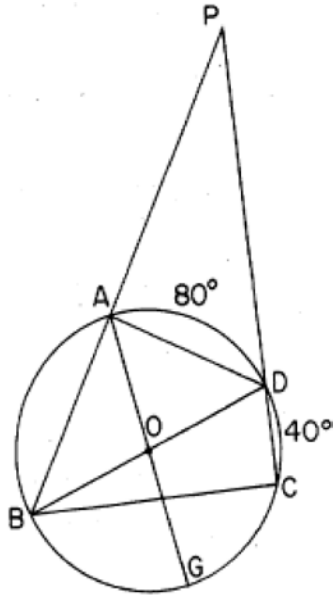
Find:  $m\widehat{BC}$ ,  $m\angle ADC$ ,  $m\angle AEB$ ,  $m\angle ADP$ ,  $m\angle P$

- 4 In circle  $O$ ,  $\overline{MN}$  is a tangent,  $\overline{NP}$  is a diameter,  $\overline{MQ}$  is a secant,  $\overline{OS}$  is a radius,  $m\widehat{QN} = 160$ , and  $m\angle PNS = 40$ .



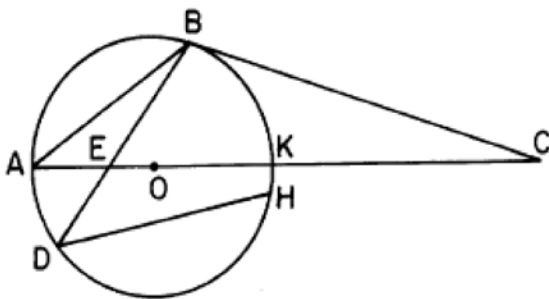
Find  $m\widehat{QP}$ ,  $m\widehat{PS}$ ,  $m\angle QRP$ ,  $m\angle NOS$ , and  $m\angle M$

- 5 Quadrilateral  $ABCD$  is inscribed in circle  $O$ ,  $\overline{BD}$  and  $\overline{AG}$  are diameters,  $\overline{PAB}$  and  $\overline{PDC}$  are secants,  $m\widehat{AD} = 80$ , and  $m\widehat{DC} = 40$ .



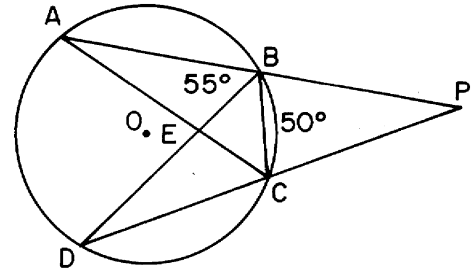
Find  $m\widehat{AB}$ ,  $m\angle BCD$ ,  $m\angle BOG$ ,  $m\angle P$ , and  $m\angle BAG$

- 6 Given: circle  $O$  with  $m\widehat{AD} : m\widehat{AB} : m\widehat{BK} = 1 : 3 : 2$ , diameter  $\overline{AK}$  is extended to  $C$ ,  $\overline{BC}$  is tangent to circle  $O$  at  $B$ , and  $m\widehat{HK} = 12^\circ$ .



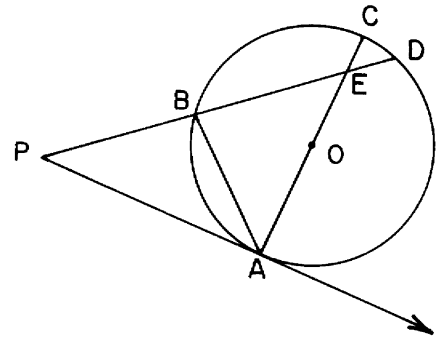
Find:  $m\widehat{AD}$ ,  $m\angle BCK$ ,  $m\angle BDH$ ,  $m\angle AEB$ ,  $m\angle DBC$

- 7 In the accompanying diagram of circle  $O$ ,  $\overline{PBA}$  and  $\overline{PCD}$  are secants, chords  $\overline{AC}$  and  $\overline{BD}$  intersect at  $E$ ,  $\overline{BA} \cong \overline{CD}$ , chord  $\overline{BC}$  is drawn,  $m\angle ABD = 55$ , and  $m\widehat{BC} = 50$ .



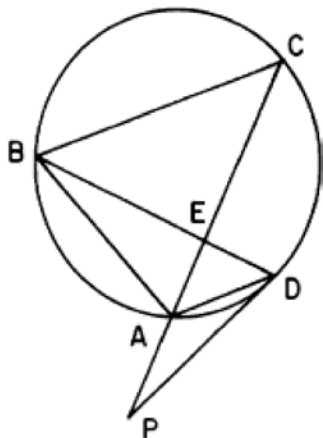
Find:  $m\angle ACD$ ,  $m\angle P$ ,  $m\angle DBC$ ,  $m\angle AED$ ,  $m\angle PCB$ .

- 8 In the accompanying diagram,  $\overrightarrow{PA}$  is a tangent to circle  $O$  at point  $A$ , secant  $\overline{PBD}$  intersects diameter  $\overline{AC}$  at point  $E$ ,  $m\angle P = 40$ , and  $m\widehat{CD} : m\widehat{DA} = 1 : 8$ .



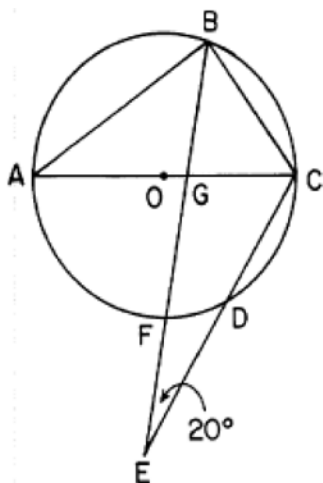
Find  $m\widehat{AD}$ ,  $m\widehat{AB}$ ,  $m\angle BEA$ ,  $m\angle BAC$ , and  $m\angle PBA$

- 9 In the accompanying diagram,  $\triangle ABC$  is isosceles with  $\overline{CB} \cong \overline{CA}$ ,  $m\angle DAC = 45^\circ$ ,  $m\widehat{BC} = 135^\circ$ ,  $\overline{PD}$  is tangent to circle  $O$  at  $D$ ,  $\overline{PAC}$  is a secant, and chords  $\overline{BD}$  and  $\overline{AC}$  intersect at  $E$ .



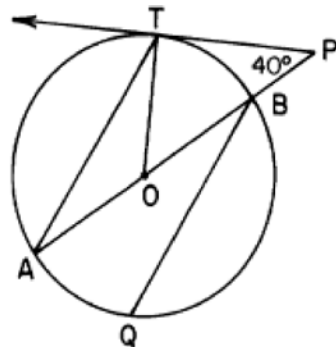
Find:  $m\widehat{AD}$ ,  $m\widehat{AB}$ ,  $m\angle P$ ,  $m\angle ADP$ ,  $m\angle BEC$

- 10 In the accompanying diagram,  $\triangle ABC$  is inscribed in circle  $O$ . Secant  $\overline{EFB}$  bisects  $\angle ABC$  and intersects diameter  $\overline{AOC}$  at  $G$ ,  $\overline{EDC}$  is a secant,  $m\angle E = 20^\circ$ , and  $m\widehat{AB} : m\widehat{BC} = 3 : 2$ .



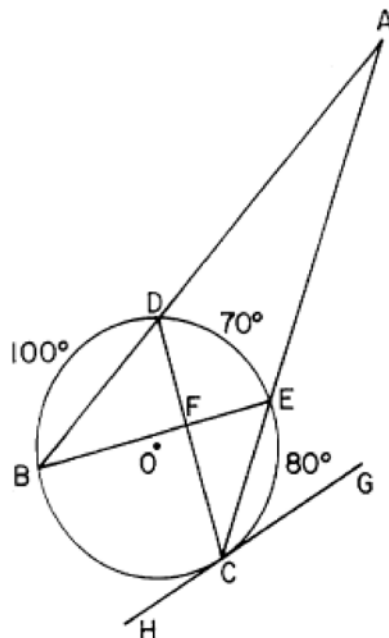
Find:  $m\widehat{BC}$ ,  $m\widehat{FD}$ ,  $m\angle ABE$ ,  $m\angle FGC$ ,  $m\angle ACD$

- 11 In the accompanying diagram of circle  $O$ ,  $\overline{PBOA}$  is a secant,  $\overline{PT}$  is tangent to circle  $O$  at  $T$ ,  $m\angle P = 40^\circ$ , and  $\overline{QB} \parallel \overline{AT}$ .



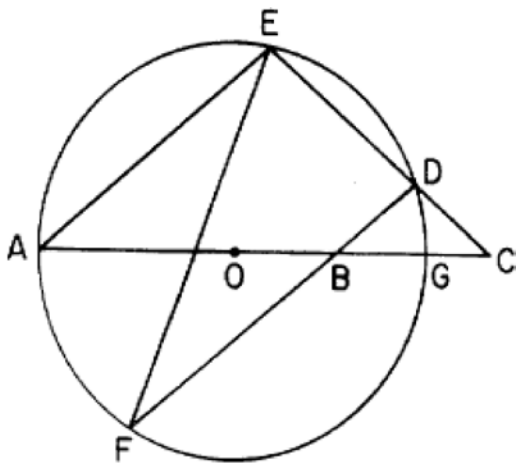
Find:  $m\angle BOT$ ,  $m\angle A$ ,  $m\widehat{AT}$ ,  $m\angle ATO$ ,  $m\angle PBQ$

- 12 In the accompanying diagram of circle  $O$ ,  $\overline{ADB}$  and  $\overline{AEC}$  are secants, chords  $\overline{BE}$  and  $\overline{CD}$  intersect at  $F$ , tangent  $\overline{GH}$  intersects circle  $O$  at  $C$ ,  $m\widehat{BD} = 100^\circ$ ,  $m\widehat{DE} = 70^\circ$ , and  $m\widehat{EC} = 80^\circ$ .



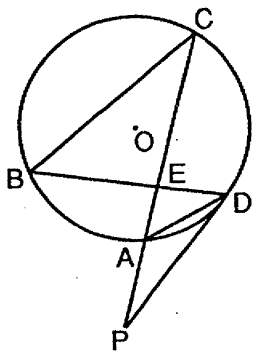
Find:  $m\angle BAC$ ,  $m\angle BDC$ ,  $m\angle CFE$ ,  $m\angle GCE$ ,  $m\angle AEB$

- 13 In the accompanying diagram of circle  $O$ ,  $\overline{AE}$  and  $\overline{FD}$  are chords,  $\overline{AOBG}$  is a diameter and is extended to  $C$ ,  $\overline{CDE}$  is a secant,  $\overline{AE} \parallel \overline{FD}$ , and  $m\widehat{AE} : m\widehat{ED} : m\widehat{DG} = 5 : 3 : 1$ .



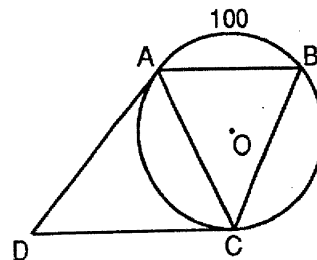
Find  $m\widehat{DG}$ ,  $m\angle AEF$ ,  $m\angle DBG$ ,  $m\angle DCA$ , and  $m\angle CDF$ .

- 14 In the accompanying diagram,  $\overline{PD}$  is tangent to circle  $O$  at  $D$ ,  $\overline{PAC}$  is a secant, chords  $\overline{BD}$  and  $\overline{AC}$  intersect at  $E$ , chord  $\overline{AD}$  is drawn,  $m\widehat{BC} = m\widehat{CA}$ ,  $m\widehat{BC}$  is twice  $m\widehat{AB}$ , and  $m\angle DAC = 48$ .



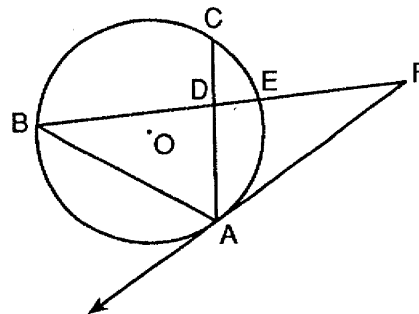
Find  $m\widehat{AB}$ ,  $m\widehat{AD}$ ,  $m\angle CPD$ ,  $m\angle CED$  and  $m\angle ADP$ .

- 15 In the accompanying diagram,  $\overline{AB} \parallel \overline{CD}$ ,  $\overline{AD}$  and  $\overline{DC}$  are tangent to circle  $O$ ,  $m\widehat{AB} = 100$ , and  $m\widehat{AC} = m\widehat{CB}$ .



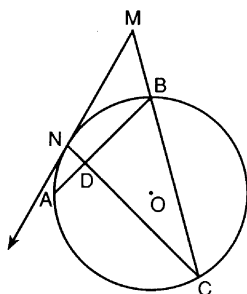
Find  $m\widehat{AC}$ ,  $m\angle B$ ,  $m\angle D$  and  $m\angle BCD$ .  
Is  $ABCD$  a parallelogram? [Explain your answer.]

- 16 In circle  $O$ ,  $\overrightarrow{FA}$  is a tangent,  $\overline{FEDB}$  is a secant,  $\overline{ADC}$  and  $\overline{AB}$  are chords,  $m\widehat{CE} = 40$ ,  $m\widehat{AB} = 130$ , and  $m\angle CAB = 60$ .



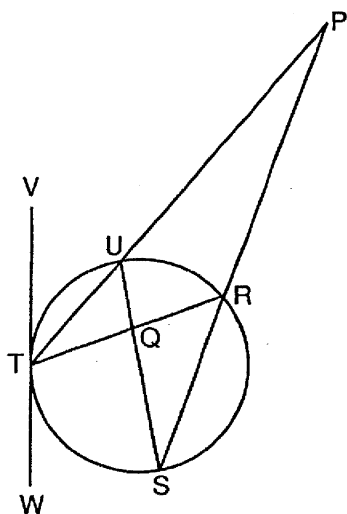
Find:  $m\widehat{BC}$ ,  $m\angle EBA$ ,  $m\angle ADE$ ,  $m\angle F$ ,  $m\angle FAC$

- 17 In the accompanying diagram of circle  $O$ , the ratio  $m\widehat{BC} : m\widehat{CA} : m\widehat{AN} : m\widehat{NB}$  is  $5 : 4 : 1 : 2$ . Chord  $\overline{CB}$  is extended to external point  $M$ , chords  $\overline{AB}$  and  $\overline{CN}$  intersect at  $D$ , and tangent  $\overrightarrow{MN}$  is drawn.



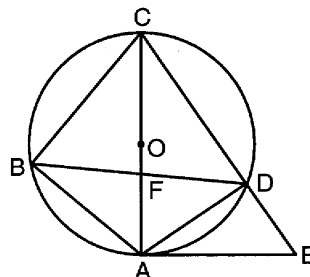
Find:  $m\widehat{BC}$ ,  $m\angle ABC$ ,  $m\angle NMC$ ,  $m\angle NDA$ ,  $m\angle MND$

- 18 In the accompanying diagram, chords  $\overline{RT}$  and  $\overline{US}$  intersect at  $Q$ , secants  $\overline{PUT}$  and  $\overline{PRS}$  are drawn,  $m\widehat{RS} = 120$ ,  $m\widehat{UT} = 80$ ,  $m\angle TRS = 50$ , and  $\overline{VW}$  is tangent to the circle at  $T$ .



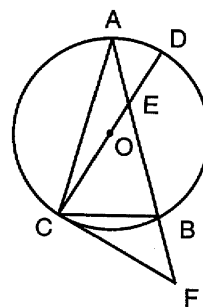
Find  $m\widehat{UR}$ ,  $m\angle SUT$ ,  $m\angle P$ ,  $m\angle RQS$  and  $m\angle PTV$ .

- 19 In the accompanying diagram of circle  $O$ , diameter  $\overline{CA}$  intersects chord  $\overline{BD}$  at  $F$ ;  $\overline{AE}$  is a tangent;  $\overline{EDC}$  is a secant,  $\overline{CB}$ ,  $\overline{BA}$ , and  $\overline{AD}$  are chords;  $m\widehat{BC} = 100$ ; and  $m\widehat{AD} = 70$ .



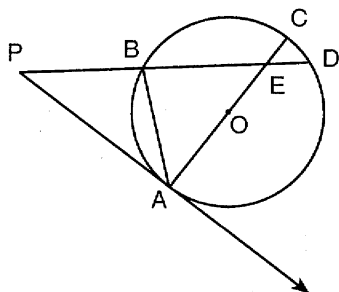
Find:  $m\widehat{AB}$ ,  $m\angle AEC$ ,  $m\angle BCA$ ,  $m\angle DFA$ ,  $m\angle DAE$ .

- 20 In the accompanying diagram of circle  $O$  with inscribed isosceles triangle  $ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $m\widehat{CB} = 60$ ,  $\overline{FC}$  is a tangent, and secant  $\overline{FBA}$  intersects diameter  $\overline{CD}$  at  $E$ .



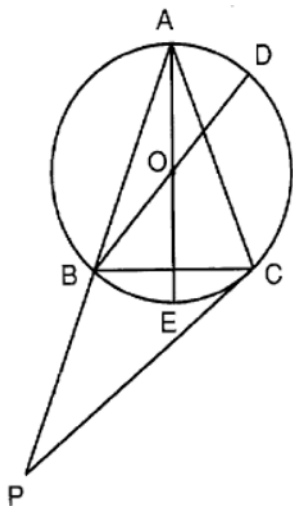
Find:  $m\angle ADC$ ,  $m\widehat{AD}$ ,  $m\angle DEB$ ,  $m\angle AFC$ ,  $m\angle BCF$

- 21 In the accompanying diagram,  $\overrightarrow{PA}$  is tangent to circle  $O$  at point  $A$ , secant  $PBD$  intersects diameter  $\overline{AC}$  at point  $E$ , chord  $\overline{AB}$  is drawn,  $m\angle P = 40$ , and  $m\widehat{CD} : m\widehat{DA} = 1 : 8$ .



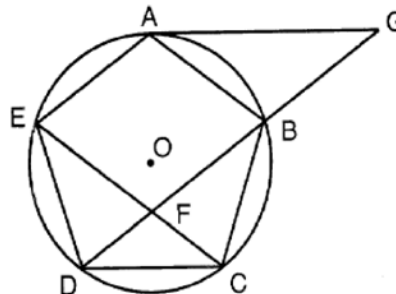
Find:  $m\widehat{DA}$ ,  $m\widehat{AB}$ ,  $m\angle BEA$ ,  $m\angle BAC$ ,  $m\angle PBA$ .

- 22 In the accompanying diagram, isosceles triangle  $ABC$  is inscribed in circle  $O$ , and vertex angle  $BAC$  measures  $40^\circ$ . Tangent  $\overline{PC}$ , secant  $\overline{PBA}$ , and diameters  $\overline{BD}$  and  $\overline{AE}$  are drawn.



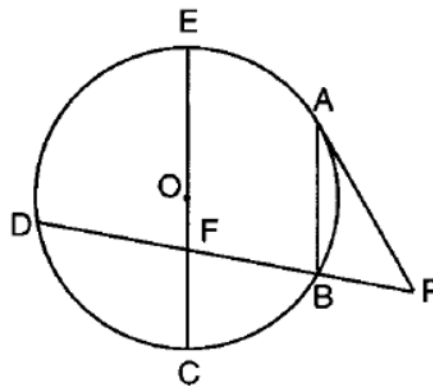
Find:  $m\widehat{BC}$ ,  $m\angle ABD$ ,  $m\angle DOE$ ,  $m\angle P$ ,  $m\angle ACP$ .

- 23 In the accompanying diagram, regular pentagon  $ABCDE$  is inscribed in circle  $O$ , chords  $\overline{EC}$  and  $\overline{DB}$  intersect at  $F$ , chord  $\overline{DB}$  is extended to  $G$ , and tangent  $\overline{GA}$  is drawn.



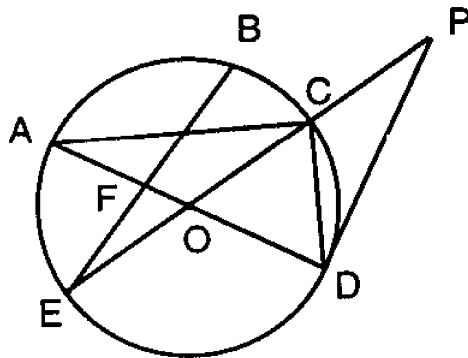
Find:  $m\angle BDE$ ,  $m\angle BFC$ ,  $m\angle AGD$

- 24 In the accompanying diagram of circle  $O$ , chord  $\overline{AB}$  is parallel to diameter  $\overline{EC}$ , secant  $\overline{PBD}$  intersects  $\overline{EC}$  at  $F$ , tangent  $\overline{PA}$  is drawn,  $m\widehat{AB} = m\widehat{BC}$ , and  $m\widehat{CD} = 80$ .



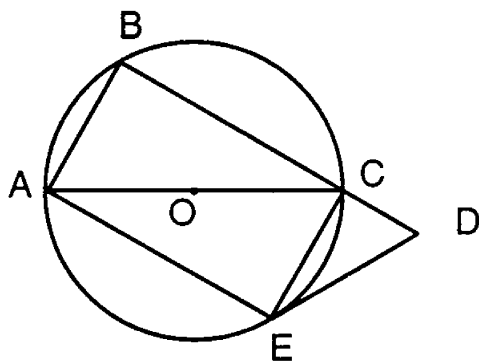
Find:  $m\widehat{AE}$ ,  $m\angle ABD$ ,  $m\angle DFC$ ,  $m\angle P$ ,  $m\angle PAB$ .

- 25 In the accompanying diagram of circle  $O$ , diameter  $\overline{EOC}$  is extended through  $C$  to point  $P$ ; diameter  $\overline{AFOD}$ , tangent  $\overline{PD}$ , and chords  $\overline{AC}$ ,  $\overline{CD}$ ,  $\overline{BFE}$  are drawn;  $m\angle COD = 60^\circ$ ; and  $m\angle AFB = 100^\circ$ .



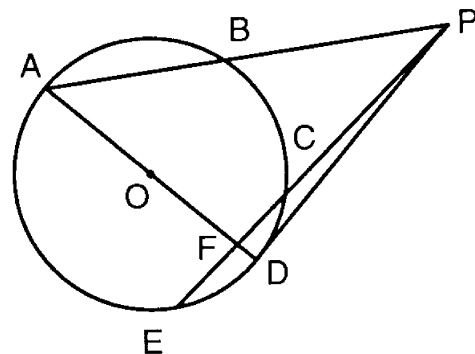
Find:  $m\widehat{DE}$ ;  $m\angle P$ ;  $m\angle ACE$ ,  $m\widehat{AB}$ ,  $m\angle ACD$ .

- 26 In the accompanying diagram of circle  $O$ ,  $m\widehat{AB} : m\widehat{BC} = 1 : 2$ ; diameter  $\overline{CA}$  and chord  $\overline{AE}$  are drawn; chord  $\overline{EC}$  is parallel to chord  $\overline{AB}$ ; chord  $\overline{BC}$  is extended through  $C$  to  $D$ ; and tangent  $\overline{DE}$  is drawn.



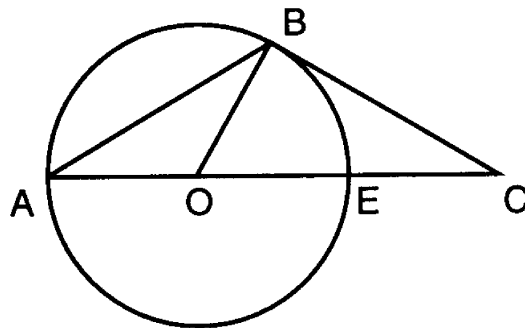
Find:  $m\widehat{BC}$ ,  $m\widehat{CE}$ ,  $m\angle AEC$ ,  $m\angle CED$ ,  $m\angle BDE$ .

- 27 In the accompanying diagram of circle  $O$ ,  $m\widehat{AC} = 140^\circ$ ,  $m\widehat{AE} = 130^\circ$ ,  $m\widehat{AB} : m\widehat{BC} = 6 : 4$ ,  $\overline{PD}$  is a tangent, secant  $\overline{PCE}$  intersects diameter  $\overline{AD}$  at  $F$ , and secant  $\overline{PBA}$  is drawn.



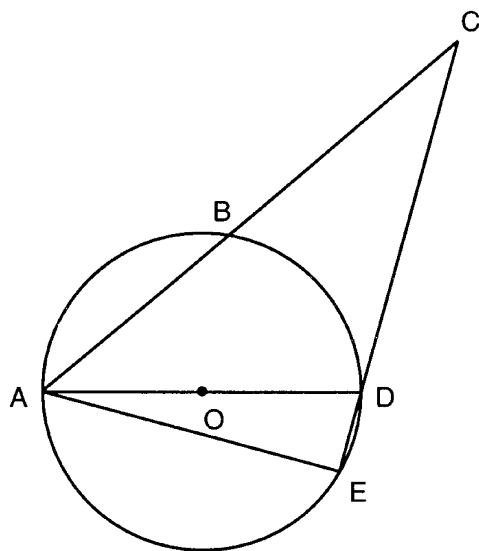
Find  $m\widehat{ED}$ ,  $m\widehat{AB}$ ,  $m\angle BAD$ ,  $m\angle APE$ ,  $m\angle EFD$

- 28 In the accompanying diagram of circle  $O$ , diameter  $\overline{AE}$  is extended through  $E$  to  $C$ ; tangent  $\overline{CB}$ , chord  $\overline{AB}$ , and radius  $\overline{OB}$  are drawn; and  $m\widehat{AB} : m\widehat{BE} = 2 : 1$ .



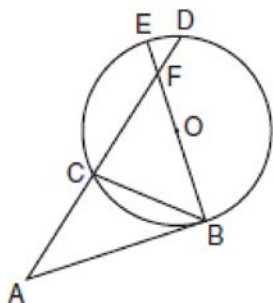
- a Find:  $m\widehat{AB}$ ,  $m\angle BAC$ ,  $m\angle C$ ,  $m\angle ABC$ .  
b Is  $\triangle OBC$  acute, right, obtuse or equiangular?  
Explain your answer.

- 29 In the accompanying diagram of circle  $O$ , diameter  $\overline{AD}$ , chord  $\overline{AE}$ , and secants  $\overline{CBA}$  and  $\overline{CDE}$  are drawn;  $m\angle BAD = 40^\circ$ ; and  $m\widehat{AE} = 5(m\widehat{ED})$ .



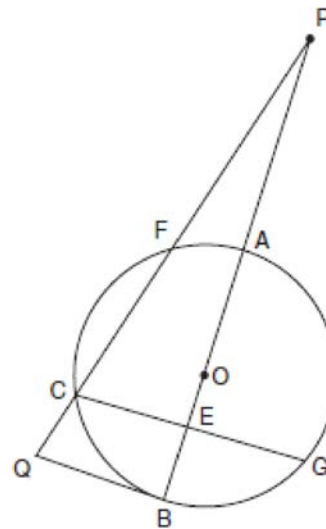
Find:  $m\widehat{BD}$ ,  $m\widehat{AE}$ ,  $m\angle ACE$ ,  $m\angle AED$ ,  $m\angle ADC$ .

- 30 In the accompanying diagram of circle  $O$ , tangent  $\overline{AB}$  and chord  $\overline{BC}$  are drawn, secant  $\overline{ACD}$  intersects diameter  $\overline{EB}$  at  $F$ ,  $m\widehat{BD} = 160$ , and  $m\widehat{BC} = 80$ .



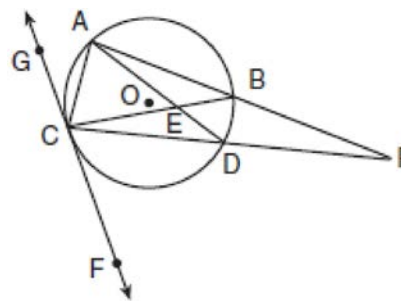
Find:  $m\angle A$ ,  $m\angle ABE$ ,  $m\angle ABC$ ,  $m\angle EFC$ ,  $m\angle ACB$

- 31 In the accompanying diagram of circle  $O$ , secant  $\overline{PFCQ}$ , secant  $\overline{PAOEB}$ , tangent  $\overline{QB}$ , and chord  $\overline{CEG}$  are drawn;  $m\widehat{BC} : m\widehat{CF} : m\widehat{FA} = 7 : 8 : 3$ ; and  $m\angle AEG = 95^\circ$ .



Find:  $m\widehat{CF}$ ,  $m\widehat{AG}$ ,  $m\angle P$ ,  $m\angle FCG$ ,  $m\angle FQB$

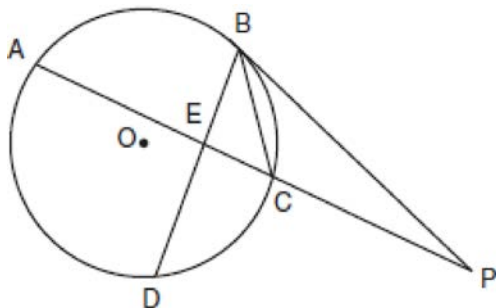
- 32 In the accompanying diagram of circle  $O$ , secant  $\overline{ABP}$ , secant  $\overline{CDP}$ , and chord  $\overline{AC}$  are drawn; chords  $\overline{AD}$  and  $\overline{BD}$  intersect at  $E$ , tangent  $\overleftrightarrow{GCF}$  intersects circle  $O$  at  $C$ , and  $m\widehat{AB} : m\widehat{BD} : m\widehat{DC} : m\widehat{CA} = 8 : 2 : 5 : 3$ .



Find:  $m\widehat{CA}$ ,  $m\angle ACB$ ,  $m\angle P$ ,  $m\angle AEB$ ,  $m\angle DCF$

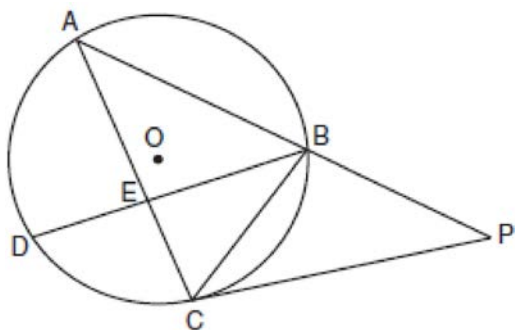


- 33 In the accompanying diagram of circle  $O$ , tangent  $\overline{PB}$ , secant  $\overline{AEC}$ , chord  $\overline{DEB}$ , and chord  $\overline{CB}$  are drawn;  $m\widehat{DC} = 90$ ;  $m\angle DEC = 85$ ;  $BP = 15$ ; and  $CB = 8$ .



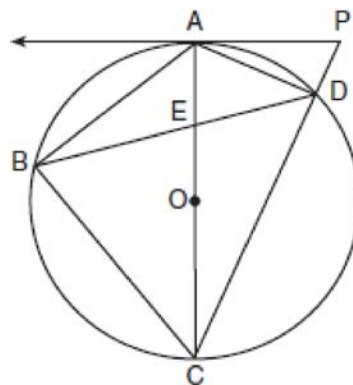
Find:  $m\widehat{AB}$ ;  $m\angle ACB$ ;  $m\angle P$  to the nearest degree.

- 34 In the accompanying diagram of circle  $O$ , chords  $\overline{BD}$ ,  $\overline{BC}$ , and  $\overline{AC}$ , tangent  $\overline{PC}$ , and secant  $\overline{ABP}$  are drawn;  $m\angle DBC = 40$ ,  $m\angle AEB = 110$ ; and  $m\widehat{AD} : m\widehat{CB} = 9 : 5$ .



Find:  $m\widehat{AB}$ ,  $m\widehat{AD}$ ,  $m\angle P$ ,  $m\angle BCP$ ,  $m\angle ACP$

- 35 In the accompanying diagram of circle  $O$ ,  $\overrightarrow{PA}$  is tangent to the circle at  $A$ ;  $\overline{PDC}$  is a secant; diameter  $\overline{AEOC}$  intersects chord  $\overline{BD}$  at  $E$ ; chords  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{DA}$  are drawn;  $m\widehat{DA} = 46$ ; and  $m\widehat{BC}$  is 32 more than  $m\widehat{AB}$ .



Find:  $m\widehat{AB}$ ;  $m\angle BAC$ ;  $m\angle P$ ;  $m\angle DEC$ ;  $m\angle PDA$

**G.G.51: Arcs Determined by Angles 7: Investigate theorems about the arcs determined by angles intersecting a circle when the vertex is on the circle**

**Answer Section**

- 1 ANS:  
40, 60, 70, 50, 60  
  
REF: 010436siii
- 2 ANS:  
90, 45, 40, 95, 30  
  
REF: 018539siii
- 3 ANS:  
60, 60, 105, 45, 60  
  
REF: 068542siii
- 4 ANS:  
20, 80, 60, 100, 30  
  
REF: 088641siii
- 5 ANS:  
100, 90, 80, 30, 40  
  
REF: 088538siii
- 6 ANS:  
36, 18, 42, 126, 108  
  
REF: 088940siii
- 7 ANS:  
55, 30, 50, 80, 75  
  
REF: 068939siii
- 8 ANS:  
160, 80, 50, 50, 100  
  
REF: 068640siii
- 9 ANS:  
45, 90, 22°30', 22°30', 90  
  
REF: 018736siii
- 10 ANS:  
72, 32, 45, 99, 61  
  
REF: 068741siii
- 11 ANS:  
50, 25, 130, 25, 155  
  
REF: 088742siii

- 12 ANS:  
20, 55, 90, 40, 125  
  
REF: 068840siii
- 13 ANS:  
20, 30, 40, 40, 100  
  
REF: 018937siii
- 14 ANS:  
72, 48, 24, 84, 24  
  
REF: 069437siii
- 15 ANS:  
130, 65, 50, 115, no, because  $\angle B$  is not congruent to  $\angle D$   
  
REF: 089439siii
- 16 ANS:  
120, 35, 95, 30, 55  
  
REF: 019537siii
- 17 ANS:  
150, 60, 45, 90, 105  
  
REF: 069537siii
- 18 ANS:  
60, 50, 20, 100, 40  
  
REF: 089537siii
- 19 ANS:  
80, 55, 40, 85, 35  
  
REF: 019639siii
- 20 ANS:  
75, 30, 135, 45, 30  
  
REF: 069636siii
- 21 ANS:  
160, 80, 50, 50, 100  
  
REF: 089636siii
- 22 ANS:  
80, 20, 140, 30, 110  
  
REF: 069737siii
- 23 ANS:  
72, 72, 36  
  
REF: 089738siii

- 24 ANS:  
60, 80, 100, 50, 30  
  
REF: 019839siii
- 25 ANS:  
120, 30, 30, 80, 90  
  
REF: 069837siii
- 26 ANS:  
120, 60, 90, 30, 60  
  
REF: 089842siii
- 27 ANS:  
50, 84, 48, 37, 95  
  
REF: 019937siii
- 28 ANS:  
120, 30, 30, 120, right because  $m\angle OBC = 90$   
  
REF: 069939siii
- 29 ANS:  
80, 150, 35, 90, 105  
  
REF: 089937siii
- 30 ANS:  
40, 90, 40, 130, 100  
  
REF: 010239siii
- 31 ANS:  
80, 120, 20, 75, 70  
  
REF: 060240siii
- 32 ANS:  
60, 80, 10, 130, 50  
  
REF: 080242siii
- 33 ANS:  
80, 40, 20  
  
REF: 010336siii
- 34 ANS:  
140, 90, 60, 25, 95  
  
REF: 060336siii
- 35 ANS:  
74, 53, 67, 104, 90  
  
REF: 080338siii