F.IF.C.7: Graphing Polynomial Functions

1. A 4th degree polynomial has zeros $-5, 3, i,$ and $-i$. Which graph could represent the function defined by this polynomial?

2. If $a, b,$ and $c$ are all positive real numbers, which graph could represent the sketch of the graph of $p(x) = -a(x + b) \left( x^2 - 2cx + c^2 \right)$?
3. Which graph has the following characteristics?
   - three real zeros
   - as \( x \to -\infty \), \( f(x) \to -\infty \)
   - as \( x \to \infty \), \( f(x) \to \infty \)

4. The zeros of a quartic polynomial function are 2, \(-2\), 4, and \(-4\). Use the zeros to construct a possible sketch of the function, on the set of axes below.

5. On the axes below, sketch a possible function
   \[ p(x) = (x - a)(x - b)(x + c), \]
   where \( a \), \( b \), and \( c \) are positive, \( a > b \), and \( p(x) \) has a positive y-intercept of \( d \). Label all intercepts.
6. On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and -2.

7. The zeros of a quartic polynomial function \( h \) are \(-1, \pm 2, \) and 3. Sketch a graph of \( y = h(x) \) on the grid below.

8. On the grid below, graph the function \( f(x) = x^3 - 6x^2 + 9x + 6 \) on the domain \(-1 \leq x \leq 4\).

9. Graph \( y = x^3 - 4x^2 + 2x + 7 \) on the set of axes below.
10 Find algebraically the zeros for 
\[ p(x) = x^3 + x^2 - 4x - 4. \] On the set of axes below, graph \( y = p(x) \).

11 A major car company analyzes its revenue, \( R(x) \), and costs \( C(x) \), in millions of dollars over a fifteen-year period. The company represents its revenue and costs as a function of time, in years, \( x \), using the given functions.

\[
R(x) = 550x^3 - 12,000x^2 + 83,000x + 7000
\]
\[
C(x) = 880x^3 - 21,000x^2 + 150,000x - 160,000
\]
The company's profits can be represented as the difference between its revenue and costs. Write the profit function, \( P(x) \), as a polynomial in standard form. Graph \( y = P(x) \) on the set of axes below over the domain \( 2 \leq x \leq 16 \).

Over the given domain, state when the company was the least profitable and the most profitable, to the nearest year. Explain how you determined your answer.
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Answer Section

1 ANS: 2 REF: 061816a

2 ANS: 1
The zeros of the polynomial are at \(-b\) and \(c\). The sketch of a polynomial of degree 3 with a negative leading coefficient should have end behavior showing as \(x\) goes to negative infinity, \(f(x)\) goes to positive infinity. The multiplicities of the roots are correctly represented in the graph.

REF: spr1501a

3 ANS: 3
The graph shows three real zeros, and has end behavior matching the given end behavior.

REF: 061604a

4 ANS:

REF: 011926a

5 ANS:

REF: 081732a

6 ANS:

REF: 011729a
7 \text{ANS:}

\begin{center}
\begin{tikzpicture}
\draw[help lines] (-3,0) grid (3,3);
\draw (-3,0) -- (3,0);
\draw (0,-3) -- (0,3);
\end{tikzpicture}
\end{center}

REF: 011831aii

8 \text{ANS:}

\begin{center}
\begin{tikzpicture}
\draw[help lines] (-3,0) grid (3,3);
\draw (-3,0) -- (3,0);
\draw (0,-3) -- (0,3);
\end{tikzpicture}
\end{center}

REF: 061826aii

9 \text{ANS:}

\begin{center}
\begin{tikzpicture}
\draw[help lines] (-3,0) grid (3,3);
\draw (-3,0) -- (3,0);
\draw (0,-3) -- (0,3);
\end{tikzpicture}
\end{center}

REF: 012032aii
ANS:

\[ 0 = x^2(x + 1) - 4(x + 1) \]

\[ 0 = (x^2 - 4)(x + 1) \]

\[ 0 = (x + 2)(x - 2)(x + 1) \]

\[ x = -2, -1, 2 \]

REF: 081633aii

ANS:

\[ P(x) = R(x) - C(x) = -330x^3 + 9000x^2 - 67000x + 167000 \]

Least profitable at year 5 because there is a minimum in \( P(x) \). Most profitable at year 13 because there is a maximum in \( P(x) \).

REF: 081837aii