

## Lesson 12-7: Counting Methods and Permutations

### Part 1: Using the Multiplication Counting Principle

1. 060728a, P.I. A2.S.9  
Max goes through the cafeteria line and counts seven different meals and three different desserts that he can choose. Which expression can be used to determine how many different ways Max can choose a meal and a dessert?  
[A]  $7 \cdot 3$  [B]  ${}_7P_3$   
[C]  $7! \cdot 3!$  [D]  ${}_7C_3$
2. 010612a, P.I. A2.S.9  
Robin has 8 blouses, 6 skirts, and 5 scarves. Which expression can be used to calculate the number of different outfits she can choose, if an outfit consists of a blouse, a skirt, and a scarf?  
[A]  $8 + 6 + 5$  [B]  $8 \cdot 6 \cdot 5$   
[C]  ${}_{19}C_3$  [D]  $8!6!5!$
3. 060607a, P.I. A2.S.9  
Leo purchased five shirts, three pairs of pants, and four pairs of shoes. Which expression represents how many different outfits consisting of one shirt, one pair of pants, and one pair of shoes Leo can make?  
[A]  ${}_{12}C_3$  [B]  $5 \cdot 3 \cdot 4$   
[C]  ${}_{12}P_3$  [D]  $5 + 3 + 4$
4. 080704a, P.I. A2.S.9  
Jen and Barry's ice cream stand has three types of cones, six flavors of ice cream, and four kinds of sprinkles. If a serving consists of a cone, one flavor of ice cream, and one kind of sprinkles, how many different servings are possible?  
[A]  ${}_{13}P_3$  [B] 72 [C]  ${}_{13}C_3$  [D] 90
5. 060403a, P.I. A.N.7  
How many different outfits consisting of a hat, a pair of slacks, and a sweater can be made from two hats, three pairs of slacks, and four sweaters?  
[A] 29 [B] 9 [C] 12 [D] 24
6. 080204a, P.I. A.N.7  
Juan has three blue shirts, two green shirts, seven red shirts, five pairs of denim pants, and two pairs of khaki pants. How many different outfits consisting of one shirt and one pair of pants are possible?  
[A] 19 [B] 420 [C] 84 [D] 130
7. 089923a, P.I. A.N.7  
Paloma has 3 jackets, 6 scarves, and 4 hats. Determine the number of different outfits consisting of a jacket, a scarf, and a hat that Paloma can wear.
8. 080404a, P.I. A.N.7  
The school cafeteria offers five sandwich choices, four desserts, and three beverages. How many different meals consisting of one sandwich, one dessert, and one beverage can be ordered?  
[A] 12 [B] 60 [C] 3 [D] 1

9. 010503a, P.I. A.N.7  
A deli has five types of meat, two types of cheese, and three types of bread. How many different sandwiches, consisting of one type of meat, one type of cheese, and one type of bread, does the deli serve?  
[A] 75 [B] 30 [C] 10 [D] 25
10. 080502a, P.I. A.N.7  
Cole's Ice Cream Stand serves sixteen different flavors of ice cream, three types of syrup, and seven types of sprinkles. If an ice cream sundae consists of one flavor of ice cream, one type of syrup, and one type of sprinkles, how many different ice cream sundaes can Cole serve?  
[A] 26 [B] 3 [C] 336 [D] 10,836
11. 010405a, P.I. A.N.7  
In a school building, there are 10 doors that can be used to enter the building and 8 stairways to the second floor. How many different routes are there from outside the building to a class on the second floor?  
[A] 18 [B] 80 [C] 10 [D] 1
12. 060501a, P.I. A.N.7  
Jeremy's bedroom has two doors leading into the hallway. His house has four doors leading to the outside. Using the doorways, in how many different ways can Jeremy leave his room and go outside?  
[A] 4 [B] 6 [C] 5 [D] 8
13. 080111a, P.I. A.N.7  
A certain car comes in three body styles with a choice of two engines, a choice of two transmissions, and a choice of six colors. What is the minimum number of cars a dealer must stock to have one car of every possible combination?  
[A] 13 [B] 36 [C] 72 [D] 42
14. 080636a, P.I. A.N.7  
Debbie goes to a diner famous for its express lunch menu. The menu has five appetizers, three soups, seven entrees, six vegetables, and four desserts. How many different meals consisting of either an appetizer *or* a soup, one entree, one vegetable, and one dessert can Debbie order?
15. 010218a  
When Kimberly bought her new car, she found that there were 72 different ways her car could be equipped. Her choices included four choices of engine and three choices of transmission. If her only other choice was color, how many choices of color did she have?  
[A] 65 [B] 6 [C] 60 [D] 12
- Part 2: Finding Permutations
16. 080107a, P.I. A.N.6  
The value of  $5!$  is  
[A] 5 [B] 120 [C] 20 [D]  $\frac{1}{5}$

17. 080503a, P.I. A.N.6  
The value of  $\frac{7!}{3!}$  is  
[A] 7 [B] 840 [C] 24 [D] 4
18. 060605a, P.I. A.N.6  
What is the value of  $\frac{8!}{4!}$ ?  
[A] 2! [B] 4! [C] 2 [D] 1,680
19. 010713a, A2.S.10  
Which value is equivalent to  ${}_3P_3$ ?  
[A] 3! [B] 1 [C] 9 [D] 27
20. 089917a, P.I. A.N.8  
How many different 6-letter arrangements can be formed using the letters in the word "ABSENT," if each letter is used only once?  
[A] 6 [B] 720 [C] 36 [D] 46,656
21. 010013a, P.I. A.N.8  
How many different 4-letter arrangements can be formed using the letters of the word "JUMP," if each letter is used only once?  
[A] 4 [B] 24 [C] 12 [D] 16
22. 060723a, P.I. A.N.8  
What is the total number of different four-letter arrangements that can be formed from the letters in the word "VERTICAL," if each letter is used only once in an arrangement?  
[A] 40,320 [B] 1,680  
[C] 8 [D] 6,720
23. 080727a, P.I. A2.S.10  
Which expression represents the number of different 8-letter arrangements that can be made from the letters of the word "SAVANNAH" if each letter is used only once?  
[A] 8! [B]  ${}_8P_5$  [C]  $\frac{8!}{3!2!}$  [D]  $\frac{8!}{5!}$
24. 010829a, P.I. A2.S.10  
What is the total number of different seven-letter arrangements that can be formed using the letters in the word "MILLION"?  
[A] 30 [B] 2,520 [C] 1,260 [D] 210
25. 010114a, P.I. A.N.8  
A locker combination system uses three digits from 0 to 9. How many different three-digit combinations with no digit repeated are possible?  
[A] 30 [B] 720 [C] 1,000 [D] 504
26. 060016a, P.I. A.N.8  
How many different five-digit numbers can be formed from the digits 1, 2, 3, 4, and 5 if each digit is used only once?  
[A] 20 [B] 120 [C] 60 [D] 24
27. 060023a, P.I. A.N.8  
All seven-digit telephone numbers in a town begin with 245. How many telephone numbers may be assigned in the town if the last four digits do *not* begin or end in a zero?

28. 080616a, P.I. A.N.8  
Julia has four different flags that she wants to hang on the wall of her room. How many different ways can the flags be arranged in a row?  
[A] 24      [B] 16      [C] 1      [D] 10
29. 010323a, P.I. A.N.8  
Six members of a school's varsity tennis team will march in a parade. How many different ways can the players be lined up if Angela, the team captain, is always at the front of the line?
30. 060125a, P.I. A.N.8  
There were seven students running in a race. How many different arrangements of first, second, and third place are possible?
31. 080034a, P.I. A.N.8  
The telephone company has run out of seven-digit telephone numbers for an area code. To fix this problem, the telephone company will introduce a new area code. Find the number of new seven-digit telephone numbers that will be generated for the new area code if both of the following conditions must be met:  
o The first digit cannot be a zero or a one.  
o The first three digits cannot be the emergency number (911) or the number used for information (411).
32. 010435a, P.I. A.N.8  
In Jackson County, Wyoming, license plates are made with two letters (*A* through *Z*) followed by three digits (0 through 9). The plates are made according to the following restrictions:  
o the first letter must be *J* or *W*, and the second letter can be any of the 26 letters in the alphabet  
o no digit can be repeated  
How many different license plates can be made with these restrictions?
33. 060329a, P.I. A.N.8  
A certain state is considering changing the arrangement of letters and numbers on its license plates. The two options the state is considering are:  
Option 1: three letters followed by a four-digit number with repetition of both letters and digits allowed  
Option 2: four letters followed by a three-digit number without repetition of either letters or digits  
[Zero may be chosen as the first digit of the number in either option.]  
Which option will enable the state to issue more license plates? How many *more* different license plates will that option yield?

[1] A

[2] B

[3] B

[4] B

[5] D

[6] C

[2] 72 and an appropriate method, such as  $3 \times 6 \times 4$ , is shown.

[1] 72 and no explanation is given.

or [1] An appropriate method is shown, but the student has one computational mistake or an incomplete listing, such as 2 of the 3 clothing categories.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[7] incorrect procedure.

[8] B

[9] B

[10] C

[11] B

[12] D

[13] C

[3] 1,344, and appropriate work is shown, such as  $8 \cdot 7 \cdot 6 \cdot 4$ .

[2] Appropriate work is shown, but one computational error is made.

[1] Appropriate work is shown, but two or more computational errors are made.

or [1] Appropriate work is shown, but one conceptual error is made, such as basing the answer on ordering an appetizer and a soup, using  $5 \cdot 3 \cdot 7 \cdot 6 \cdot 4$ .

or [1] 1,344, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously

[14] incorrect procedure.

[15] B

[16] B

[17] B

[18] D

[19] A

[20] B

[21] B

[22] B

[23] C

[24] C

[25] B

[26] B

[2] 8,100 and appropriate work is shown, such as  $9 \times 10 \times 10 \times 9$ .

[1] 10,000 but appropriate work is shown.

or [1] Appropriate work is shown, but the student multiplies incorrectly.

or [1] An appropriate pattern is shown, such as  $9 \times 10 \times 10 \times 9$ .

or [1] 8,100 but no work is shown.

[0] 38 is shown.

or [0] The student attempts to use the counting principle, but adds.

or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an

[27] obviously incorrect procedure.

[28] A

- [2] 120, and appropriate work is shown, such as  $1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ .
- [1] Appropriate work is shown, but one computational error is made.  
or [1] 720 and  ${}_6P_6$  or  $6!$  is shown.  
or [1] 120, but no work is shown.  
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- 
- [29] [2] 210, and appropriate work is shown, such as  $7 \cdot 6 \cdot 5$  or  ${}_7P_3$ .
- [1] Appropriate work is shown, but no answer or an incorrect answer is found.  
or [1] 210, but no work is shown.  
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- 
- [30] [4]  $7.98 \times 10^6$  or 7,980,000 and appropriate work is shown, such as  $8 \times 10^6 - 2 \times 10^4$ .
- [3] Appropriate work is shown, but one computational error is made.  
or [3] The student uses 1–9 instead of 0–9 as the number of digits in  $8 \times 9^6 - 2 \times 9^4$ .
- [2] The student correctly produces only one part,  $8 \times 10^6$  or  $2 \times 10^4$ , but carries the process to an appropriate result.  
or [2] Appropriate work is shown, but more than one error is made.  
[1] The student produces only one part,  $8 \times 9^6$  or  $2 \times 9^4$ .  
or [1] 7,980,000 but no work is shown.  
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- 
- [31] [2] 37,440 and appropriate work is shown, such as  $2 \times 26 \times 10 \times 9 \times 8$  or  ${}_2P_1 \times {}_{26}P_1 \times {}_{10}P_1 \times {}_9P_3$ .
- [1] Appropriate work is shown, but one computational or conceptual error is made.  
or [1] Appropriate work is shown for at least one restriction, such as  $2 \times 26$  or  $10 \times 9 \times 8$ .  
or [1] 37,440 but no work is shown.  
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- 
- [32] [3] Option 2 will yield 82,576,000 more possibilities, and appropriate work is shown, such as  $26^3 \cdot 10^4$  and  ${}_{26}P_4 \cdot {}_{10}P_3$ .
- [2] Appropriate work is shown, but one computational error is made, but the appropriate option is identified.  
or [2] The correct numbers of arrangements are found for both Option 1 and Option 2, but the question of which option will yield more arrangements is not answered or is answered incorrectly.  
[1] Appropriate work is shown, but more than one computational error is made, but the appropriate option is identified.  
or [1] Appropriate work is shown, but one conceptual error is made, but the appropriate option is identified.  
or [1] Either Option 1 or Option 2 is found correctly, but no further correct work is shown.  
or [1] Option 2 will yield 82,576,000 more possibilities, but no work is shown.  
[0] Option 2, but no work or inappropriate work is shown.  
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- 
- [33] [2] 37,440 and appropriate work is shown, such as  $2 \times 26 \times 10 \times 9 \times 8$  or  ${}_2P_1 \times {}_{26}P_1 \times {}_{10}P_1 \times {}_9P_3$ .
- [1] Appropriate work is shown, but one computational or conceptual error is made.  
or [1] Appropriate work is shown for at least one restriction, such as  $2 \times 26$  or  $10 \times 9 \times 8$ .  
or [1] 37,440 but no work is shown.  
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- 

- [32] [3] Option 2 will yield 82,576,000 more possibilities, and appropriate work is shown, such as  $26^3 \cdot 10^4$  and  ${}_{26}P_4 \cdot {}_{10}P_3$ .
- [2] Appropriate work is shown, but one computational error is made, but the appropriate option is identified.  
or [2] The correct numbers of arrangements are found for both Option 1 and Option 2, but the question of which option will yield more arrangements is not answered or is answered incorrectly.  
[1] Appropriate work is shown, but more than one computational error is made, but the appropriate option is identified.  
or [1] Appropriate work is shown, but one conceptual error is made, but the appropriate option is identified.  
or [1] Either Option 1 or Option 2 is found correctly, but no further correct work is shown.  
or [1] Option 2 will yield 82,576,000 more possibilities, but no work is shown.  
[0] Option 2, but no work or inappropriate work is shown.  
or [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- 
- [33] [2] 37,440 and appropriate work is shown, such as  $2 \times 26 \times 10 \times 9 \times 8$  or  ${}_2P_1 \times {}_{26}P_1 \times {}_{10}P_1 \times {}_9P_3$ .
- [1] Appropriate work is shown, but one computational or conceptual error is made.  
or [1] Appropriate work is shown for at least one restriction, such as  $2 \times 26$  or  $10 \times 9 \times 8$ .  
or [1] 37,440 but no work is shown.  
[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
-