The University of the State of New York

260TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Thursday, June 21, 1934 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I and five questions from part II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and reduced to its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

ADVANCED ALGEBRA

Thursday, June 21, 1934

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each question has 2½ credits assigned to it; no partial credit should be allowed. Each answer must be reduced to its simplest form.

1 Express with a real denominator the sum of the fractions $\frac{2}{1-i}$ and $\frac{3}{1+i}$ *Ans*.....

2 Solve for x the following equation: $sx^2 + tx - p = 0$ *Ans*.....

3 Express the discriminant of the quadratic equation $2x^2 - kx - k = 0$ as a function of k. *Ans*.....

4 What is the sum of the roots of the equation $2x^3 - x + 1 = 0$? *Ans*.....

5 Transform the equation $x^3 - 4x - 5 = 0$ into an equation whose roots are less by 2 than the roots of the given equation. *Ans*.....

6 Transform the equation $3x^2 + 5x - 2 = 0$ into an equation with integral coefficients, that of the highest-degree term being unity. *Ans*.....

7 What is the equation of the straight line which passes through the point (2, 4) and whose slope is $\frac{1}{2}$? *Ans*.....

8 In the equation $2^x = 18$, find the value of x correct to the *nearest tenth*. *Ans*.....

9 How many signals can be displayed with three flags of different colors by showing the flags any number at a time on a vertical line? *Ans*.....

10 The records of a certain insurance company of New York show that, of 100,000 children 10 years of age, 38,569 live to be 70. What is the probability that a child of 10 will live to reach the age of 70? *Ans*.....

11 When drawn on the same set of axes, what is the number of points in which the graph of $y = mx$ can intersect the graph of $x^2 + y^2 = r^2$ if m and r are constants? *Ans*.....

12 Given the equation $5x^3 + 16x^2 + 13x + 2 = 0$; find the rational root that lies between 0 and -1. *Ans*.....

13 What is the exact number of complex roots of the equation $x^5 + x^2 + 1 = 0$? *Ans*.....

14 If ${}_nC_r$ represents the number of combinations of n things taken r at a time and n is 6, what value of r will make ${}_nC_r$ a maximum? *Ans*.....

15 The resistance (r) of a copper wire of fixed diameter, at constant temperature, varies directly as the length (l). If 180 ohms is the resistance of a wire 2880 feet long, determine the linear function connecting r and l. *Ans*.....

16 The graph of $y = ax^2 + c$, where a and c are constants, cuts the x-axis in two points whose abscissas are numerically equal but opposite in sign. What is the equation of the axis of symmetry? *Ans*.....

17 Is the function $x\sqrt{2} + 3x = 7$ rational or irrational? *Ans.*.....

18 Write $x = \log_b a$ as an exponential equation. *Ans.*.....

19 If the complex numbers $1 + i$ and $2i$ are represented graphically by the points A and B respectively and their sum is represented by the point C, how many degrees are there in the angle ACB? *Ans.*.....

20 The geometric mean of x and y is 4. If this statement is expressed in the form of an equation, what is the name of the graph that pictures this relationship between x and y? *Ans.*.....

ADVANCED ALGEBRA

Thursday, June 21, 1934

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.
The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

In the examination in advanced algebra the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

21 What must be the value of k in the equation $x^3 - 3x^2 - 6x + k = 0$ if the roots of the equation are in arithmetic progression? [10]

22 Find, correct to the *nearest tenth*, the positive root of the equation
$x^3 + 2x^2 + 5x - 14 = 0$ [10]

23 The number (n) of vibrations of an iron wire of diameter 0.2 centimeters and length l centimeters, under a tension of w grams, is given by the formula $n = \frac{1}{2l}\sqrt{\frac{980w}{.077\pi}}$; using logarithms, find, correct to the *nearest integer*, the number of vibrations for such a wire when $l = 25$ centimeters and $w = 520$ grams. [Use $\pi = 3.142$] [10]

24 A chemist wishes to prepare a 40% solution of alcohol. He adds pure alcohol to 50 cubic centimeters of 15% alcohol but on testing the resulting mixture finds that it is 50% alcohol. How much water must he then add in order to obtain a mixture of the desired strength? [10]

25 It is said that the rectangle most pleasing to the human eye is that in which the sum of the two dimensions is to the longer as the longer is to the shorter. Using this principle, find, correct to the *nearest inch*, the dimensions of a page in a book if its area is 40 square inches. [5, 5]

26 Prove that if $a + bi$ is a root of a rational integral equation of the nth degree with real coefficients, then $a - bi$ is also a root. [10]

27 *a* Using the same set of axes, draw the graph of (1) $x^2 - y^2 = 10$ from $x = -6$ to $x = +6$ inclusive, (2) $2y - x = 2$ [8]

b From the graphs made in answer to *a*, find, correct to the *nearest tenth*, the coordinates of the points of intersection of the two graphs. [2]

*28 A point is moving on a straight line in such a manner that its distance (s) in feet from the origin, at any time (t) in seconds, is given by the formula $s = 2t^3 - 15t^2 + 24t + 72$

a Find the rate in feet per second at which the point is moving when $t = 5$ [4]

b When does the point reverse its motion? [3]

c What is the minimum distance between the point and the origin for positive values of t? [3]

*29 The table below shows the number of deaths from scarlet fever in New York State during a certain year and the ages at which they occurred.

Age	1	2	3	4	5	6	7	8	9	10	11	12	13
Number of deaths	9	26	30	32	30	25	17	6	3	3	2	2	1

a From the above data draw a frequency curve. [6]

b Is the distribution symmetrical? [1]

c What age represents most nearly the *median* number of deaths? [2]

d At what age does the *mode* occur? [1]

* This question is based on one of the optional topics in the syllabus.