The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Tuesday, January 26, 1993 — 9:15 a.m. to 12:15 p.m., only

Notice . . .

Calculators must be available to all students taking this examination.

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

The "Reference Tables for Mathematics" and a formula sheet which you may need to answer some questions in this examination are stapled in the center of this booklet. Open the booklet and carefully remove the reference tables.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer 30 questions from this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may left in terms of π or in radical form. [60]

1 In acute triangle ABC, $a = 3$, $b = 4$, and $\sin A = 0.3$. What is the value of $\sin B$?

2 Express in simplest form:

$$\frac{x^2 - 9}{2x^2 + 5x - 3} \div \frac{1}{2x - 1}$$

3 Express the sum of $4 + \sqrt{-36}$ and $-2 - \sqrt{-49}$ in $a + bi$ form.

4 Find the value of $\sum_{k=2}^{5} (k - 1)^3$.

5 Express 240° in radian measure.

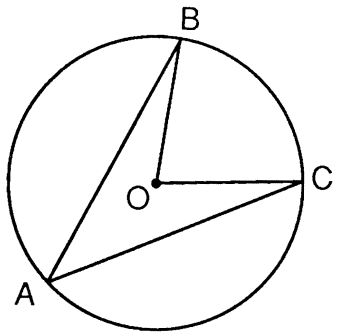
6 What is the positive value of $\sin x$ that satisfies the equation $\sin^2 x + 4 \sin x - 5 = 0$?

7 If $3^x = \frac{1}{9}$, what is the value of x?

8 Express $\tan 240°$ as a function of a positive acute angle.

9 Solve for x: $\sqrt{2x + 3} = x$

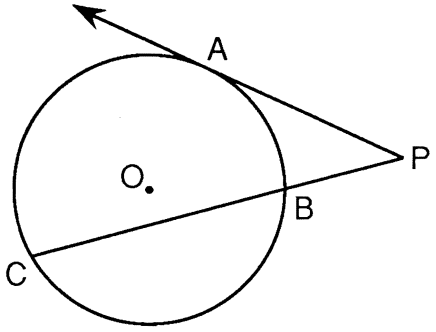
10 In the accompanying diagram of circle O, the measure of inscribed angle BAC is 40°. Find, in degrees, the measure of central angle BOC.



11 If $f(x) = 3x - 1$ and $g(x) = x^2 + 1$, evaluate $(g \circ f)(-1)$.

12 In $\triangle RST$, $r = 8$, $s = 10$, and $m\angle T = 120$. Express the area of $\triangle RST$ in radical form.

13 In the accompanying diagram, $\overrightarrow{PA}$ is tangent to circle O at A and $\overline{PBC}$ is a secant. If $\overset{\frown}{AB}:\overset{\frown}{BC}:\overset{\frown}{CA} = 2:3:4$, find $m\angle P$.

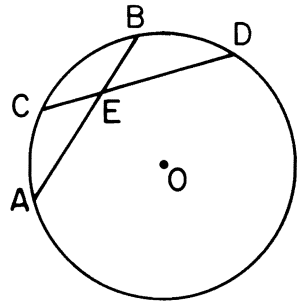


14 How many distinct triangles may be constructed if $a = 4$, $b = 5$, and $m\angle A = 30$?

15 If $\cos (2x + 25)° = \sin 35°$, find x.

Directions (16–35): For *each* question chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

16 In the accompanying diagram of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $AE \times EB = 18$ and $ED = 6$, what is CE?



(1) 108
(2) 24
(3) 3
(4) 18

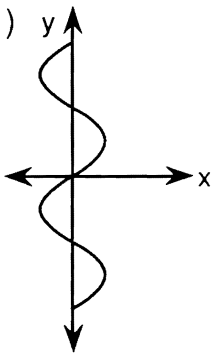
17 If the coordinates of A are (3,5), what are the coordinates of $(r_{x\text{-axis}} \cdot D_2)(A)$?

(1) (-6,10)
(2) (6,-10)
(3) (-10,6)
(4) (-6,-10)

18 If $f(x) = 8^x$, what is the value of $f\left(\frac{1}{3}\right)$?

(1) $\frac{1}{2}$
(2) 2
(3) $\frac{8}{3}$
(4) 4

19 Which diagram shows a relation that is *not* a function?

(1)


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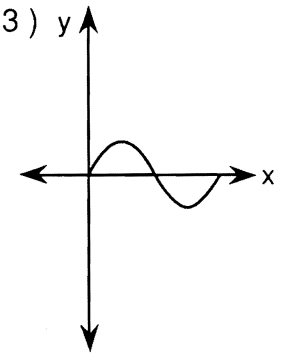

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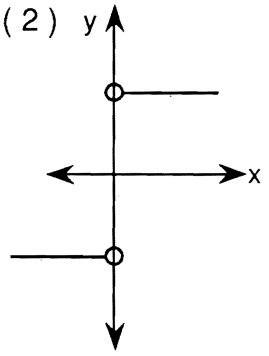

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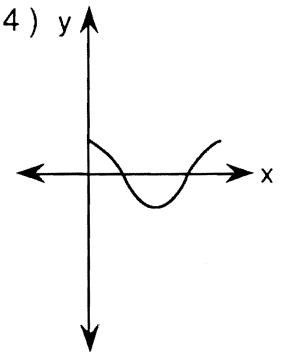


20 The expression $\frac{\sec \theta}{\tan \theta}$ is equivalent to

(1) $\sin \theta$
(2) $\cos \theta$
(3) $\sec \theta$
(4) $\csc \theta$

21 The value of $\cos\left(\text{Arc tan } \frac{8}{15}\right)$ is

(1) $\frac{8}{17}$
(2) $-\frac{8}{17}$
(3) $\frac{15}{17}$
(4) $\frac{\sqrt{161}}{15}$

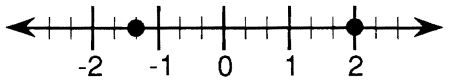
22 If $\log 5 = a$, then $\log 0.0005$ is

(1) $3 - a$
(2) $a - 3$
(3) $4 - a$
(4) $a - 4$

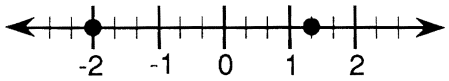
23 If a fair coin is tossed four times, the probability of four heads is

(1) $\frac{1}{16}$
(2) $\frac{2}{16}$
(3) $\frac{3}{16}$
(4) $\frac{4}{16}$

24 Which is a graph of the solution set of $|3x + 1| = 5$?

(1)


(2)


(3)


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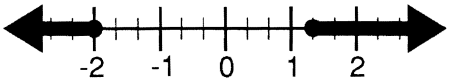
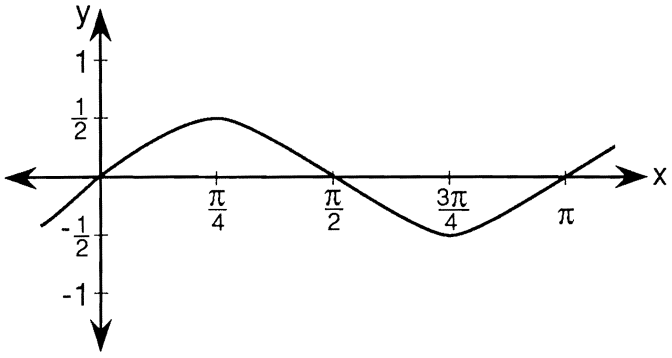


25 If the graph of the equation $y = x^2$ is reflected in the line whose equation is $y = x$, an equation of the image is

(1) $x = -y^2$
(2) $x = y^2$
(3) $y = -x^2$
(4) $y = \log x^2$

26 Which equation is represented by the graph below?



(1) $y = 2 \sin \frac{1}{2}x$
(2) $y = \frac{1}{2} \sin \frac{1}{2}x$
(3) $y = \frac{1}{2} \sin 2x$
(4) $y = -\frac{1}{2} \cos 2x$

27 For which value of x is $\tan (x + 30)°$ undefined?

(1) -30
(2) 60
(3) 150
(4) 330

28 If $(\sqrt{18} + \sqrt{2})$ is divided by $\sqrt{2}$, the result is

(1) 16
(2) $\sqrt{10}$
(3) 3
(4) 4

29 Cos $2A + 1$ is equivalent to

(1) $2\cos^2 A$
(2) $2\sin^2 A$
(3) $\cos^2 A + 1$
(4) $2 \sin A \cos A + 1$

30 Which equation has roots of $5 - 2i$ and $5 + 2i$?

(1) $x^2 - 10x + 29 = 0$
(2) $x^2 - 10x - 21 = 0$
(3) $x^2 + 10x - 21 = 0$
(4) $x^2 + 10x + 29 = 0$

31 The inequality $\sin \theta \geq \cos \theta$ is true for all values of θ in the interval

(1) $0° \leq \theta \leq 90°$
(2) $0° \leq \theta \leq 360°$
(3) $45° \leq \theta \leq 225°$
(4) $45° \leq \theta \leq 315°$

32 A given set of scores forms a normal distribution. The mean of the set of scores is 80 and the standard deviation is 6. What percentage of the scores lie between 80 and 86?

(1) 34
(2) 47.5
(3) 68
(4) 95

33 The expression $\log a + \frac{1}{2} \log b$ is equivalent to

(1) $\log \sqrt{ab}$
(2) $\log a\sqrt{b}$
(3) $\log (a + \sqrt{b})$
(4) $\left(\log a\right)\left(\frac{1}{2} \log b\right)$

34 The graph of $xy = 10$ is best described as

(1) an ellipse
(2) two intersecting lines
(3) a parabola
(4) a hyperbola

35 Which set of numbers is *not* closed with respect to the given operation?

(1) integers with respect to multiplication
(2) even integers with respect to addition
(3) integers with respect to subtraction
(4) odd integers with respect to addition

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Calculations that may be obtained by mental arithmetic or the calculator do not need to be shown. [40]

36 *a* Sketch the graph of the equation $y = 2 \cos x$ in the interval $0 \leq x \leq \pi$. Label your answer *a*. [4]

b On the same set of axes, reflect the graph drawn in part *a* in the *x*-axis. Label the graph *b*. [3]

c On the same set of axes, sketch the graph drawn in part *a* after a dilation $D_{\frac{1}{2}}$. Label the graph *c*. [3]

37 *a* Solve for *x*: $\dfrac{5}{x-3} - \dfrac{30}{x^2-9} = 1$ [5]

b Solve for *x* and express the roots in simplest $a \pm bi$ form:

$$9x^2 - 30x + 34 = 0$$ [5]

38 Given the following scores on a math test:

70, 75, 70, 85, 90, 90, 85, 80, 90, 70,
80, 75, 90, 80, 75, 70, 80, 75, 90, 70

a Find:

(1) the standard deviation to the *nearest hundredth* [6]

(2) the number of scores that fall outside one standard deviation from the mean [2]

b One more student takes the same test. The mean of all the scores is now exactly 80. What is this student's score? [2]

39 *a* Using logarithms, calculate $\sqrt[3]{0.972}$ to the *nearest hundredth*. [5]

b In a family of five children, what is the probability that *at most* two of the children are boys? [Assume $P(\text{boy}) = \frac{1}{2}$ and $P(\text{girl}) = \frac{1}{2}$.] [5]

40 Find, to the *nearest ten minutes*, all values of *x* in the interval $0° \leq x < 360°$ that satisfy the equation $\cos 2x - \sin^2 x + \sin x + 1 = 0$. [10]

41 A side of rhombus *ABCD* measures 100 feet. The measure of $\angle ABC = 110° 20'$.

a Find, to the *nearest foot*, the measure of diagonal $\overline{AC}$. [7]

b Find, to the *nearest square foot*, the area of rhombus *ABCD*. [3]

42 *a* For all values of *A* for which the expressions are defined, prove the following is an identity:

$$\frac{1 + \cos A + \cos 2A}{\sin A + \sin 2A} = \cot A$$ [5]

b Find, in simplest form, the middle term in the expansion of $\left(x^2 + \frac{1}{2x}\right)^8$. [5]

FOR TEACHERS ONLY

SCORING KEY

THREE-YEAR SEQUENCE FOR HIGH SCHOOL MATHEMATICS

COURSE III

Tuesday, January 26, 1993 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following. [If more than 30 are answered, only the first 30 answered should be considered.] Allow no partial credit. For questions 16–35, allow credit if the pupil has written the correct answer instead of the numeral 1, 2, 3, or 4.

(1) 0.4

(2) $x - 3$

(3) $2 - i$

(4) 100

(5) $\frac{4\pi}{3}$

(6) 1

(7) -2

(8) tan 60° *or* cot 30°

(9) 3

(10) 80

(11) 17

(12) $20\sqrt{3}$

(13) 40

(14) 2

(15) 15

(16) 3

(17) 2

(18) 2

(19) 1

(20) 4

(21) 3

(22) 4

(23) 1

(24) 3

(25) 2

(26) 3

(27) 2

(28) 4

(29) 1

(30) 1

(31) 3

(32) 1

(33) 2

(34) 4

(35) 4

[OVER]

Part II

Please refer to the Department publication *Guide for Rating Regents Examinations in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Note: In October 1991, a supplement to the *Guide for Rating Regents Examinations in Mathematics* was sent to all schools. This supplement includes references to problem solving and calculator use. Teachers should become familiar with these modifications before rating student papers.

(37) *a* 2 [5]

b $\frac{5}{3} \pm i$ [5]

(38) *a* (1) 7.57 [6]

(2) 10 [2]

b 90 [2]

(39) *a* 0.99 [5]

b $\frac{16}{32}$ [5]

(40) 90°, 221°50′, 318°10′ [10]

(41) *a* 164 [7]

b 9377 [3]

(42) *b* $\frac{35}{8}x^4$ [5]