The University of the State of New York

305TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, January 27, 1949 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 Find to the *nearest degree* the smallest positive value of x which satisfies the equation $\cos^2 x - 2 \sin^2 x + \frac{1}{2} \sin x = 0$ [10]

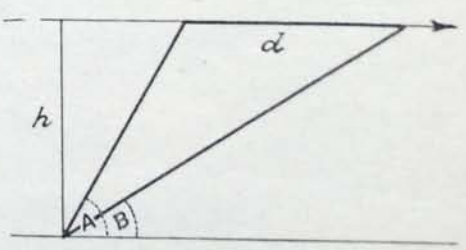
22 *a* Starting with the formula for $\cos (x + y)$, derive the formula for $\cos 2x$ in terms of $\sin x$. [5]

b Prove the identity: $\sin^2 x + \sin^2 x \tan^2 x = \tan^2 x$ [5]

23 *a* Using the same set of axes, sketch the graphs of $y = 2 \cos x$ and $y = \sin x$ as x varies from 0 to 2π radians. [5, 3]

b From the graphs made in answer to *a*, determine the number of values of x between 0 and 2π radians that will satisfy the equation $\sin x = 2 \cos x$ [2]

24 An airplane is flying horizontally due east at a height of h feet. From a certain point, its angle of elevation is A and 1 minute later its angle of elevation is B, as shown in the figure. Assuming that the observer and the line of flight are in the same vertical plane, derive an expression in terms of A, B and h for the distance d covered during the 1 minute of flight. [10]



Part III

Answer at least two questions from part III.

25 $ABCD$ is a parallelogram; side $AB = 329$, side $AD = 578$ and diagonal $AC = 627$. Find angle ABC to the *nearest degree*. [10]

26 A boat is traveling due east towing an artillery target 2000 feet behind it. At a given instant the direction of the target from a gun on the shore is N 21° 40′ E and the direction of the boat from the gun is N 38° 10′ E. Find to the *nearest foot* the distance of the gun from the target. [10]

27 In order to find the distance between two points, A and B, separated by a swamp, a station C was chosen and the distances CA and CB were found to be 350 yards and 380 yards respectively. Angle ACB was found to be 62°. Find to the *nearest yard* the distance from A to B. [10]

28 A boat sights a signal tower which has two lights 60 feet apart, one directly above the other. The angle of elevation of the lower light is 14° and that of the upper light is 28°. Find to the *nearest foot* the distance from the boat to the tower. [10]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Express tan 255° as a function of a positive acute angle. 1.....

2 Find the value of $\tan \frac{\pi}{4}$ 2.....

3 Find the logarithm of .03928 3.....

4 Find the number whose logarithm is 1.8043 4.....

5 If log cot $A = 9.4799 - 10$, find the acute angle A to the *nearest minute*. 5.....

6 Find the value of tan 44° 27′ 6.....

7 If the vertex angle of an isosceles triangle is 64° and each leg is 10 inches, find to the *nearest tenth of an inch* the length of the altitude to the base. 7.....

8 In triangle ABC, the sides are 2, 4 and 5. Find the *cosine* of the smallest angle. 8.....

9 In triangle ABC, if $b = 40$, $\sin A = .6$ and $\sin B = .8$, find a. 9.....

10 In triangle ABC, $a = 5$, $c = 4$ and angle $B = 60°$. Find the value of $\tan \frac{1}{2}(A - C)$. [Answer may be left in radical form.] 10.....

11 What is the maximum value of $\cos 3A$? 11.....

12 Using the data: angle $B = 60°$, $b = 10$ and $c = 9$, it is possible to construct (a) only one triangle, (b) two triangles or (c) no triangle. Which is correct, a, b or c? 12.....

13 If $\cos 3x = \frac{1}{2}\sqrt{3}$, find the smallest positive value of x. 13.....

14 If $\sin A = \frac{4}{5}$, $\cos B = \frac{12}{13}$ and both A and B are positive acute angles, find the value of $\cos (A - B)$. 14.....

15 Express the area of a parallelogram in terms of the sides a, b and their included angle C. 15.....

16 If A is an acute angle, express $\sin A$ in terms of $\tan A$. 16.....

17 In a circle whose radius is 6, find the length of an arc whose central angle is 2 radians. 17.....

18 If $x = \sin^{-1} \frac{8}{17}$ and x is in the first quadrant, find $\cot x$. 18.....

19 Express $\cos 10x + \cos 6x$ as a product. 19.....

20 If x is an acute angle, which of the following is always true?
(a) $\sin (180 - x) = \cos x$, (b) $\cos (180 - x) = \cos x$,
(c) $\tan (180 - x) = -\tan x$ 20.....