The University of the State of New York

312TH HIGH SCHOOL EXAMINATION

TRIGONOMETRY

Thursday, June 21, 1951 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in trigonometry.

The minimum time requirement is four or five recitations a week for half a school year, or the equivalent.

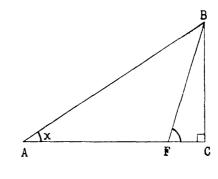*Answer five questions from parts II and III, including at least two questions from each part.*

Part II

Answer at least two questions from part II.

21 Find all values of A between $0°$ and $360°$ that satisfy the equation $3 \cos 2A + 5 \cos A + 2 = 0$. [Express approximate values of A correct to the *nearest degree*.] [10]

22 a Derive the law of cosines for the case in which the triangle is acute. [7]

b Prove the identity: $\cot A \cos A + \sin A = \csc A$ [3]

23 a Sketch the graph of $y = \sin 2x$ as x varies from 0 to 2π radians. [4]

b On the set of axes used in a, sketch the graph of $y = \sin \frac{1}{2}x$ as x varies from 0 to 2π radians. [4]

c From the graphs made in answer to a and b, determine the number of values of x (greater than 0 and less than 2π) for which $\sin 2x = \sin \frac{1}{2}x$. [2]

24 In the accompanying diagram, $AF = d$, $BF = p$, $BC = h$ and $\angle BFC = x + 45°$.

a Show that $p = \dfrac{d \sin x}{\sin 45°}$ [3]

b Show that $h = d \sin x \, (\sin x + \cos x)$ [7]

[1]

[OVER]

Part III

Answer at least two questions from part III.

25 In triangle ABC, $a = 46.5$, $b = 81.2$ and $c = 58.3$. Find B to the *nearest minute*. [10]

26 To compute the number of linear feet of pontoon bridge necessary to connect two points A and B on opposite sides of a river, a point C was chosen on the same side as A and the following measurements were made: $AC = 283$ feet, $\angle CAB = 98° 10'$ and $\angle ACB = 52° 30'$. Find to the *nearest foot* the length AB of the bridge. [4, 6]

27 An airplane leaving its base B on a course N 61° 40' E flies 86 miles to point A. Then it flies directly south 142 miles to point C. Find to the *nearest degree* the bearing of B from C. [5, 5]

28 A tower and a monument stand on a level plain. The angles of depression of the top and the bottom of the monument from the top of the tower are 41° and 56° respectively. The height of the tower is 152 feet. Find the height of the monument to the *nearest foot*. [5, 5]

TRIGONOMETRY

Fill in the following lines:

Name of pupil...Name of school...

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express in radians an angle of 162°. [Answer may be left in terms of π.]

1.....   ... .

2 In a circle whose radius is 6 inches, an arc subtends a central angle of 1.5 radians. Find the length of the arc in inches.

2....   . .

3 If $\sin A = \dfrac{1}{\sec A}$, find the smallest positive value of A.

3.....   ..

4 If $A = \sin^{-1} \dfrac{2}{\sqrt{13}}$, find the positive value of $\cos A$.

4...   . ... .

5 Express cos 250° as a function of a positive angle less than 45°.

5...   ....

6 Find the logarithm of 2.718

6... ...   .

7 Find the number whose logarithm is 8.1946 — 10

7..   . ... .

8 Find cos 41° 13′

8..   . ... ..

9 Find, to the *nearest minute,* the positive acute angle the logarithm of whose sine is 9.7733 — 10

9   . .

10 In triangle ABC, sin $A = 0.2$. sin $B = 0.6$ and $a = 30$. Find b.

10.   ....

11 In triangle ABC, $a = 8$. $b = 10$ and $c = 12$. Find cos C.

11.   .. . .

12 In triangle ABC, $a = 16$. $b = 12$ and $C = 60°$.
Find $\tan \dfrac{A-B}{2}$. [Answer may be left in radical form.]

12. .. . .

13 In triangle ABC, $a = 8$, $b = 5$ and $C = 27°$. Find, to the *nearest integer,* the area of the triangle.

13.   . . .. .

14 If A is acute, express sin A in terms of tan A.

14....   ....

[3]

15 If cos $x = m$, express the positive value of sin $\dfrac{x}{2}$ in terms of m. 15.. . . ..

16 Express sin 50° — sin 20° in terms of the product of two functions. 16..... ....

17 Express cos $(A - B)$ in terms of the sine and the cosine of A and of B. 17.. ....

Directions (questions 18–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter a, b or c.

18 Using the data $A = 43°$, $a = 12$ and $b = 20$, it is possible to construct
(a)only one triangle (b)two different triangles (c)no triangle 18.. ...

19 Sin $2x$ — 2 sin $x = 0$ is true for (a)all values of x (b)only certain values of x (c) no values of x 19. ....

20 If $\angle x = \theta$ and $\angle y = 180° + \theta$, then the tangent and the cotangent of x are
(a)always (b)sometimes (c)never numerically equal to the same function of y. 20. ....