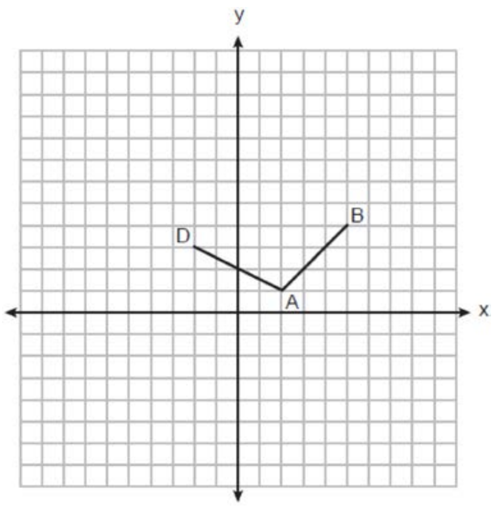
G.GPE.B.4: Quadrilaterals in the Coordinate Plane 1

1 On the set of axes below, the coordinates of three vertices of trapezoid $ABCD$ are $A(2,1)$, $B(5,4)$, and $D(-2,3)$.



Which point could be vertex C?
1) $(1,5)$
2) $(4,10)$
3) $(-1,6)$
4) $(-3,8)$

2 A quadrilateral has vertices with coordinates $(-3,1)$, $(0,3)$, $(5,2)$, and $(-1,-2)$. Which type of quadrilateral is this?
1) rhombus
2) rectangle
3) square
4) trapezoid

3 The coordinates of the vertices of parallelogram $CDEH$ are $C(-5,5)$, $D(2,5)$, $E(-1,-1)$, and $H(-8,-1)$. What are the coordinates of P, the point of intersection of diagonals $\overline{CE}$ and $\overline{DH}$?
1) $(-2,3)$
2) $(-2,2)$
3) $(-3,2)$
4) $(-3,-2)$

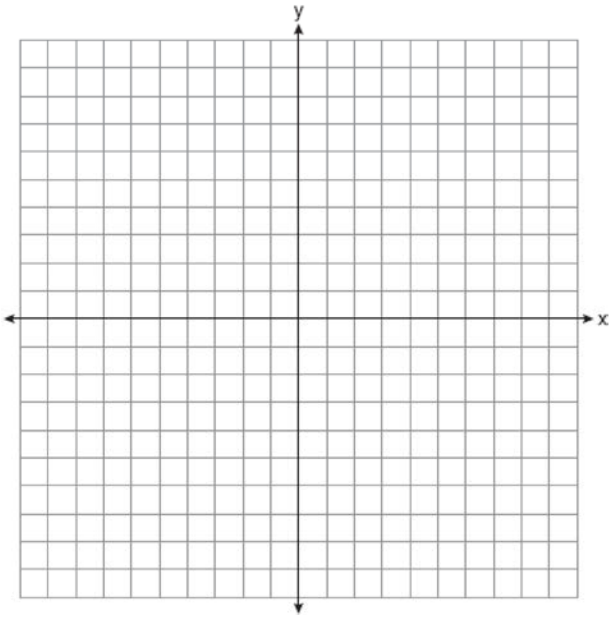
4 Rectangle $ABCD$ has two vertices at coordinates $A(-1,-3)$ and $B(6,5)$. The slope of $\overline{BC}$ is
1) $-\frac{7}{8}$
2) $\frac{7}{8}$
3) $-\frac{8}{7}$
4) $\frac{8}{7}$

5 Parallelogram $ABCD$ has coordinates $A(0,7)$ and $C(2,1)$. Which statement would prove that $ABCD$ is a rhombus?
1) The midpoint of $\overline{AC}$ is $(1,4)$.
2) The length of $\overline{BD}$ is $\sqrt{40}$.
3) The slope of $\overline{BD}$ is $\frac{1}{3}$.
4) The slope of $\overline{AB}$ is $\frac{1}{3}$.

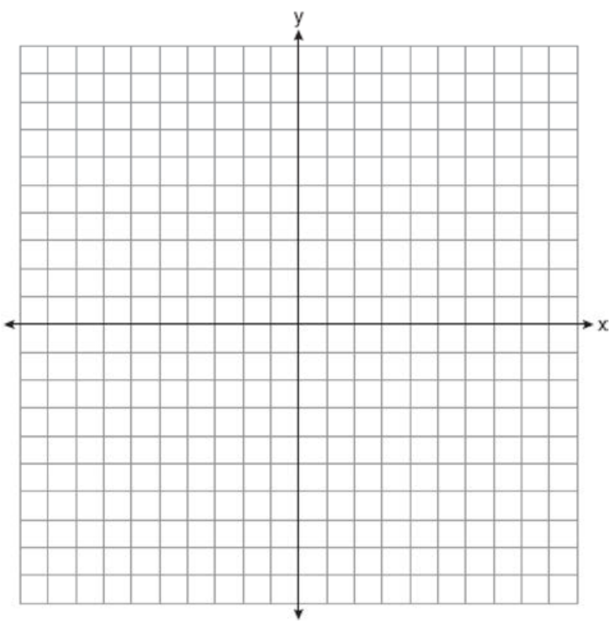
6 The diagonals of rhombus $TEAM$ intersect at $P(2,1)$. If the equation of the line that contains diagonal $\overline{TA}$ is $y=-x+3$, what is the equation of a line that contains diagonal EM?
1) $y=x-1$
2) $y=x-3$
3) $y=-x-1$
4) $y=-x-3$

Name: ________________________

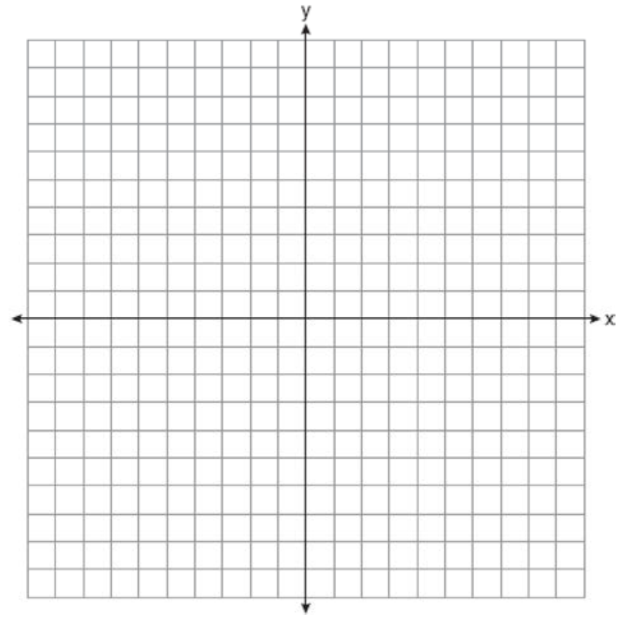
7 In square *GEOM*, the coordinates of *G* are $(2,-2)$ and the coordinates of *O* are $(-4,2)$. Determine and state the coordinates of vertices *E* and *M*. [The use of the set of axes below is optional.]



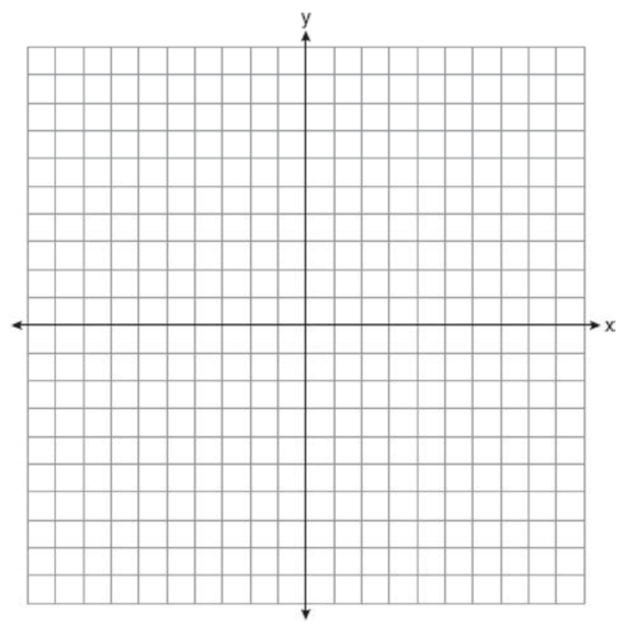
8 The coordinates of the vertices of quadrilateral *HYPE* are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$. Prove *HYPE* is a rectangle. [The use of the set of axes below is optional.]



9 Quadrilateral *NATS* has coordinates $N(-4,-3)$, $A(1,2)$, $T(8,1)$, and $S(3,-4)$. Prove quadrilateral *NATS* is a rhombus. [The use of the set of axes below is optional.]



10 Parallelogram *MATH* has vertices $M(-7,-2)$, $A(0,4)$, $T(9,2)$, and $H(2,-4)$. Prove that parallelogram *MATH* is a rhombus. [The use of the set of axes below is optional.] Determine and state the area of *MATH*.



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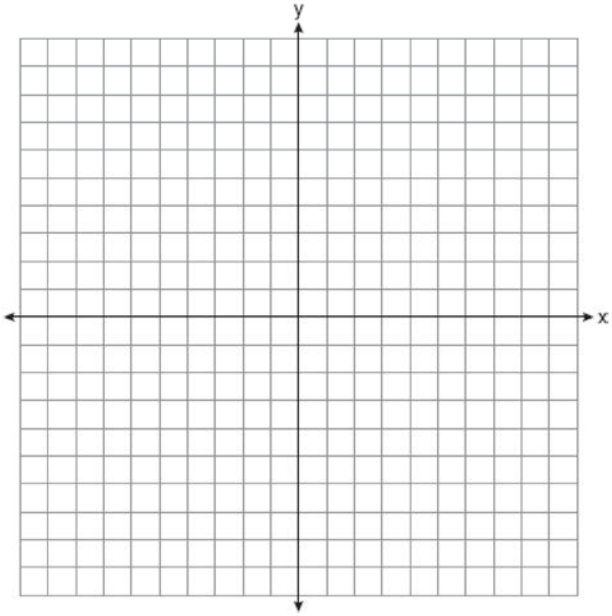
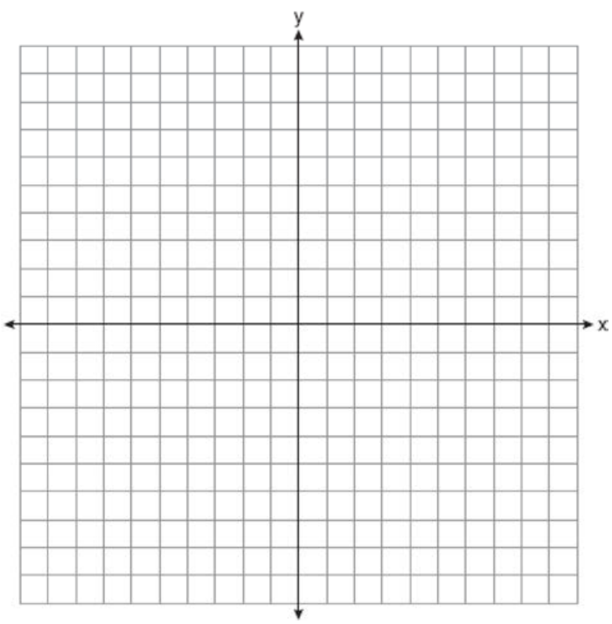
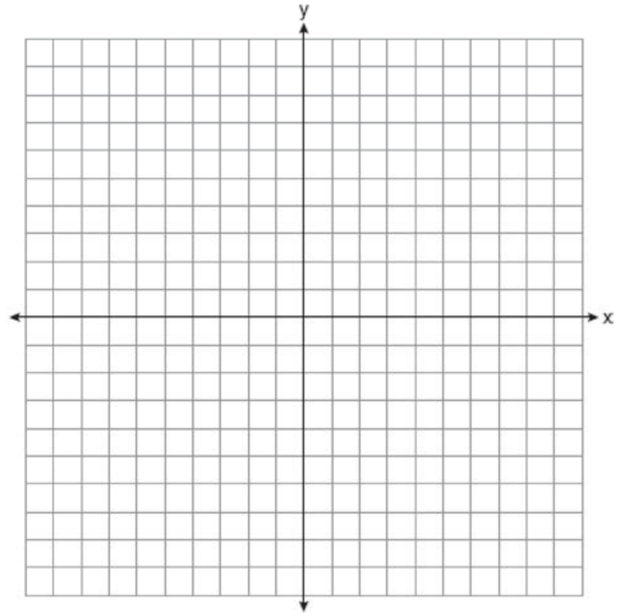
11 Quadrilateral $PQRS$ has vertices $P(-2,3)$, $Q(3,8)$, $R(4,1)$, and $S(-1,-4)$. Prove that $PQRS$ is a rhombus. Prove that $PQRS$ is *not* a square. [The use of the set of axes below is optional.]



12 The coordinates of the vertices of quadrilateral $ABCD$ are $A(0,4)$, $B(3,8)$, $C(8,3)$, and $D(5,-1)$. Prove that $ABCD$ is a parallelogram, but *not* a rectangle. [The use of the set of axes below is optional.]



13 The vertices of quadrilateral $MATH$ have coordinates $M(-4,2)$, $A(-1,-3)$, $T(9,3)$, and $H(6,8)$. Prove that quadrilateral $MATH$ is a parallelogram. Prove that quadrilateral $MATH$ is a rectangle. [The use of the set of axes below is optional.]



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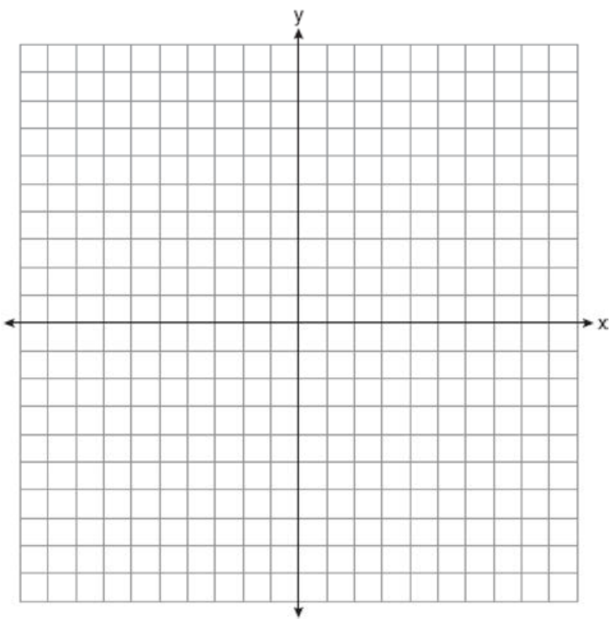
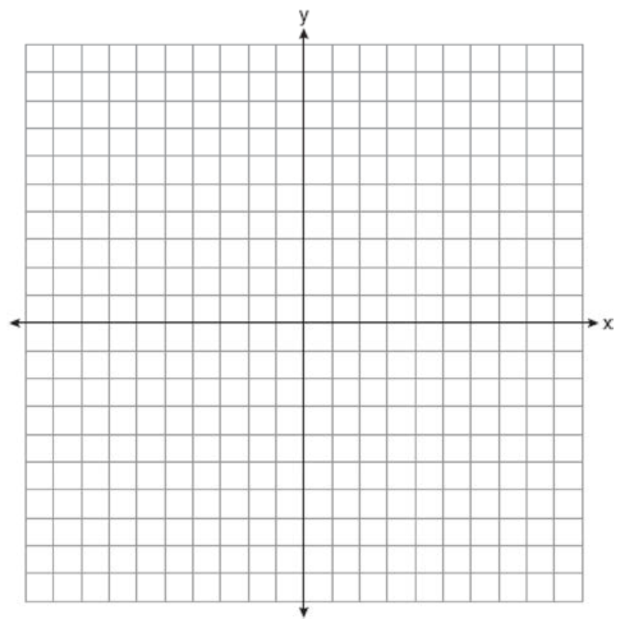
14 Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral. Prove that Riley's quadrilateral $ABCD$ is a trapezoid. [The use of the set of axes below is optional.] Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley's definition to prove that $ABCD$ is *not* an isosceles trapezoid.



15 Quadrilateral $ABCD$ has vertices with coordinates $A(-3,6)$, $B(6,3)$, $C(6,-2)$, and $D(-6,2)$. Joe defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Joe's definition to prove $ABCD$ is an isosceles trapezoid. [The use of the set of axes below is optional.]



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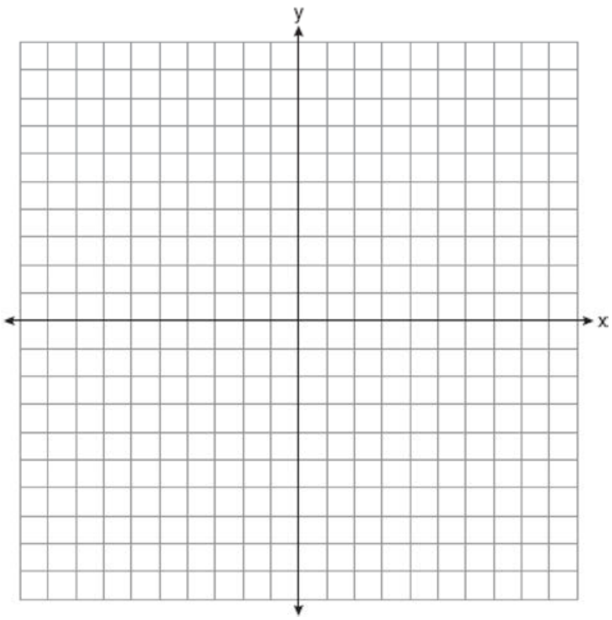
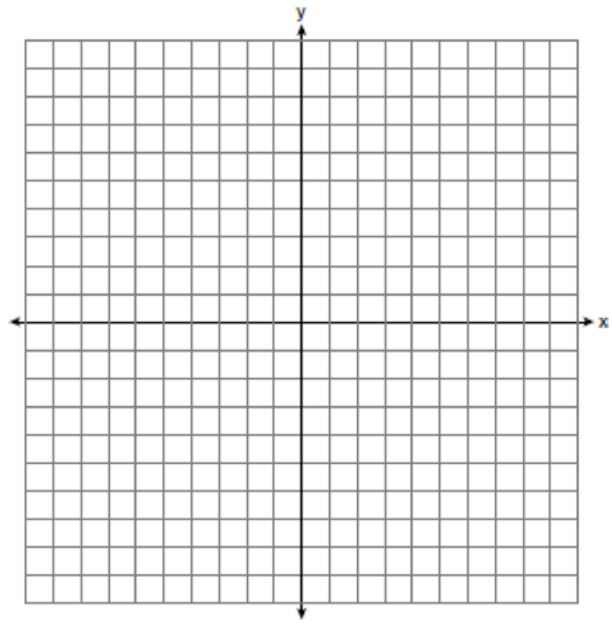
16 Quadrilateral $MATH$ has vertices with coordinates $M(-1,7)$, $A(3,5)$, $T(2,-7)$, and $H(-6,-3)$. Prove that quadrilateral $MATH$ is a trapezoid. State the coordinates of point Y such that point A is the midpoint of $\overline{MY}$. Prove that quadrilateral $MYTH$ is a rectangle. [The use of the set of axes below is optional.]



17 In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle. State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle. Prove that your quadrilateral $RSTP$ is a rectangle. [The use of the set of axes below is optional.]



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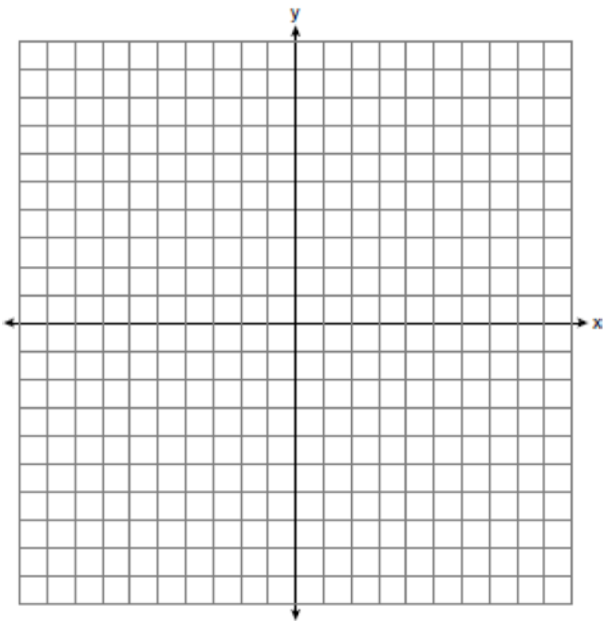
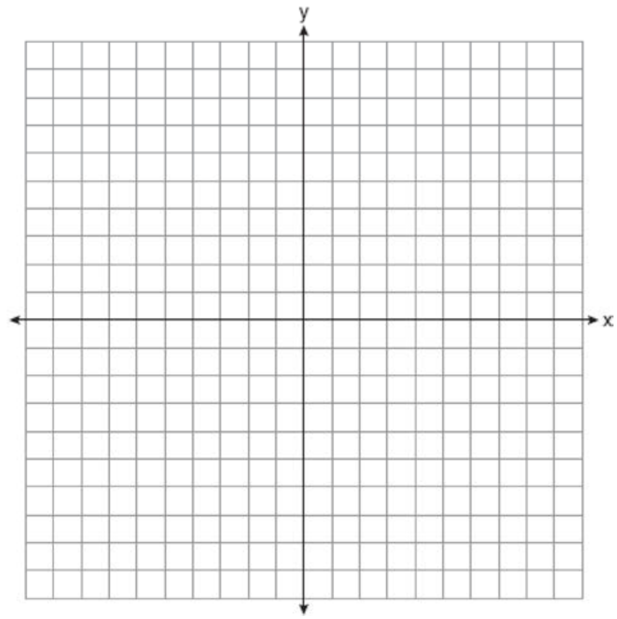
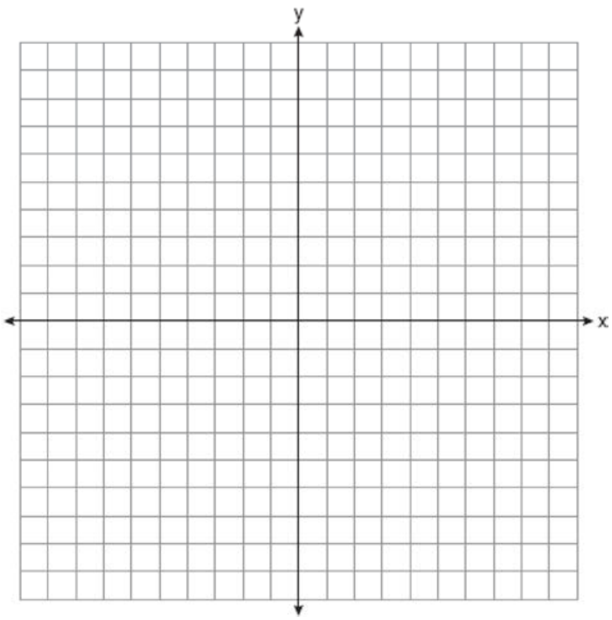
18 In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. State the coordinates of R so that quadrilateral $PART$ is a parallelogram. Prove that quadrilateral $PART$ is a parallelogram. [The use of the set of axes below is optional.]



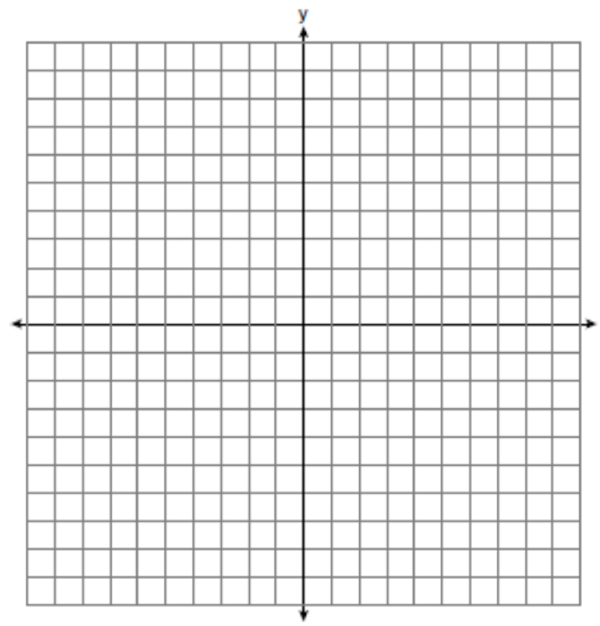
19 The coordinates of the vertices of $\triangle ABC$ are $A(1,2)$, $B(-5,3)$, and $C(-6,-3)$. Prove that $\triangle ABC$ is isosceles. State the coordinates of point D such that quadrilateral $ABCD$ is a square. Prove that your quadrilateral $ABCD$ is a square. [The use of the set of axes below is optional.]



20 The coordinates of the vertices of $\triangle ABC$ are $A(-2,4)$, $B(-7,-1)$, and $C(-3,-3)$. Prove that $\triangle ABC$ is isosceles. State the coordinates of $\triangle A'B'C'$, the image of $\triangle ABC$, after a translation 5 units to the right and 5 units down. Prove that quadrilateral $AA'C'C$ is a rhombus. [The use of the set of axes below is optional.]

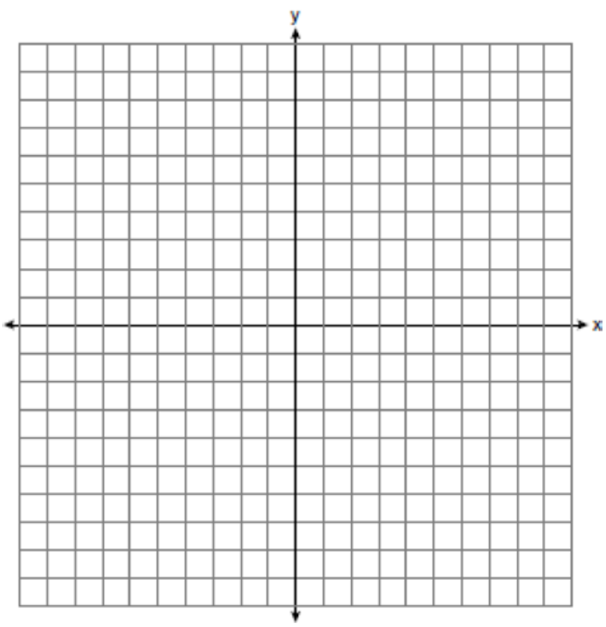


21 Given: Triangle DUC with coordinates $D(-3,-1)$, $U(-1,8)$, and $C(8,6)$
Prove: $\triangle DUC$ is a right triangle
Point U is reflected over $\overline{DC}$ to locate its image point, U', forming quadrilateral $DUCU'$.
Prove quadrilateral $DUCU'$ is a square.
[The use of the set of axes below is optional.]

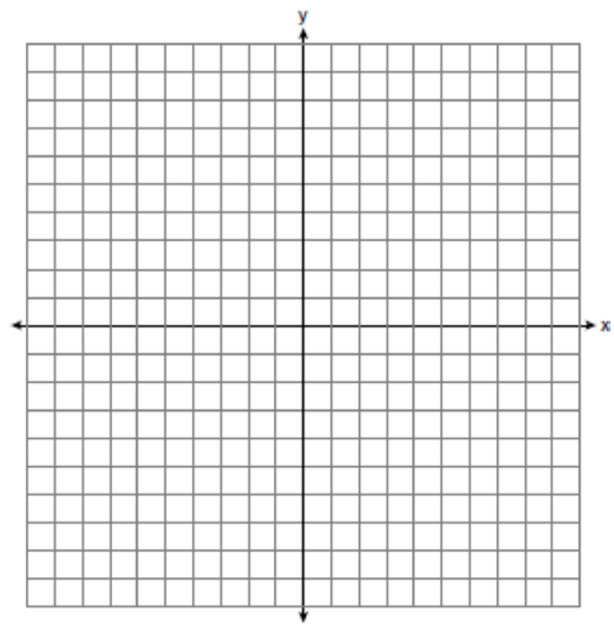


Name: ________________________

22 Triangle PET has vertices with coordinates $P(-6,4)$, $E(6,8)$, and $T(-4,-2)$. Prove $\triangle PET$ is a right triangle. State the coordinates of N, the image of P, after a 180° rotation centered at $(1,3)$. Prove $PENT$ is a rectangle. [The use of the set of axes below is optional.]



23 In rhombus $MATH$, the coordinates of the endpoints of the diagonal $\overline{MT}$ are $M(0,-1)$ and $T(4,6)$. Write an equation of the line that contains diagonal $\overline{AH}$. [Use of the set of axes below is optional.] Using the given information, explain how you know that your line contains diagonal $\overline{AH}$.



G.GPE.B.4: Quadrilaterals in the Coordinate Plane 1
Answer Section

1 ANS: 4

$m_{\overline{AD}} = \frac{3-1}{-2-2} = \frac{2}{-4} = -\frac{1}{2}$ A pair of opposite sides is parallel.

$m_{\overline{BC}} = \frac{8-4}{-3-5} = \frac{4}{-8} = -\frac{1}{2}$

REF: 082321geo

2 ANS: 4

$\frac{-2-1}{-1--3} = \frac{-3}{2}$ $\frac{3-2}{0-5} = \frac{1}{-5}$ $\frac{3-1}{0--3} = \frac{2}{3}$ $\frac{2--2}{5--1} = \frac{4}{6} = \frac{2}{3}$

REF: 081522geo

3 ANS: 3

$M_x = \frac{-5+-1}{2} = -\frac{6}{2} = -3$ $M_y = \frac{5+-1}{2} = \frac{4}{2} = 2.$

REF: 081902geo

4 ANS: 1

$m_{\overline{AB}} = \frac{-3-5}{-1-6} = \frac{-8}{-7} = \frac{8}{7}$

REF: 062315geo

5 ANS: 3

$\frac{7-1}{0-2} = \frac{6}{-2} = -3$ The diagonals of a rhombus are perpendicular.

REF: 011719geo

6 ANS: 1

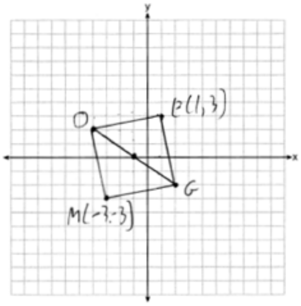
$m_{\overline{TA}} = -1$ $y = mx + b$

$m_{\overline{EM}} = 1$ $1 = 1(2) + b$

$-1 = b$

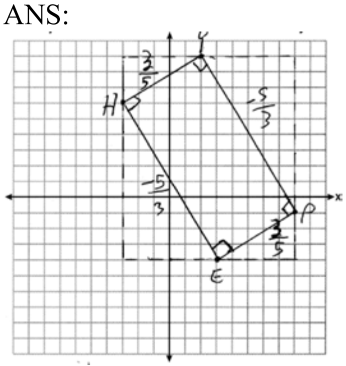
REF: 081614geo

7 ANS:



REF: 011731geo

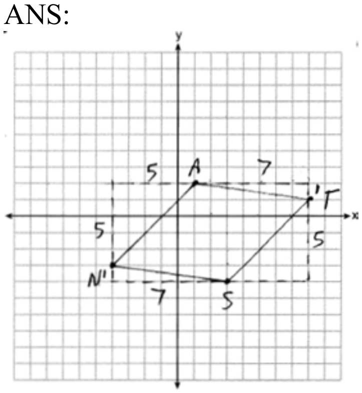
8 ANS:



1) Quadrilateral $HYPE$ with $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$ (Given); 2) Slope of $\overline{HY}$ and $\overline{PE}$ is $\frac{3}{5}$, slope of $\overline{YP}$ and $\overline{EH}$ is $-\frac{5}{3}$ (Slope determined graphically); 3) $\overline{HY} \perp \overline{YP}$, $\overline{PE} \perp \overline{EH}$, $\overline{YP} \perp \overline{PE}$, $\overline{EY} \perp \overline{HY}$ (The slopes of perpendicular lines are opposite reciprocals); 4) $\angle H$, $\angle Y$, $\angle P$, $\angle E$ are right angles (Perpendicular lines form right angles); 5) $HYPE$ is a rectangle (A rectangle has four right angles).

REF: 082233geo

9 ANS:



$$\overline{AN} \cong \overline{AT} \cong \overline{TS} \cong \overline{SN}$$

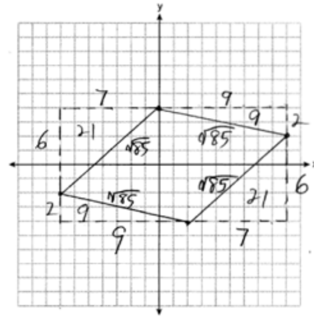
$$\sqrt{5^2+5^2} = \sqrt{7^2+1^2} = \sqrt{5^2+5^2} = \sqrt{7^2+1^2}$$

$$\sqrt{50} = \sqrt{50} = \sqrt{50} = \sqrt{50}$$

Quadrilateral $NATS$ is a rhombus because all four sides are congruent.

REF: 012032geo

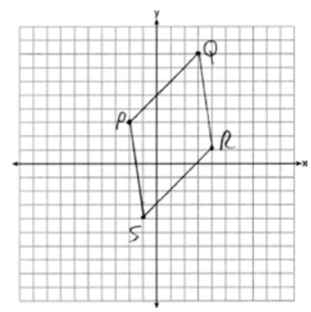
10 ANS:
A rhombus has four congruent sides. Since each side measures $\sqrt{85}$, all four sides of *MATH* are congruent, and



MATH is a rhombus. $16 \times 8 - (21 + 9 + 21 + 9) = 68$

REF: 062334geo

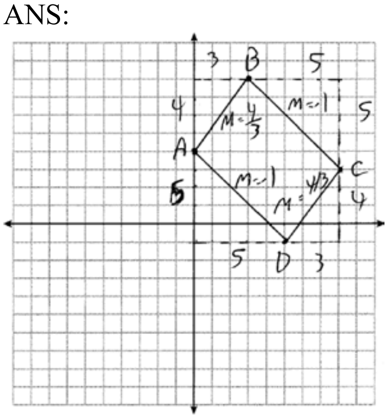
11 ANS:
$\overline{PQ}$ $\sqrt{(8-3)^2 + (3--2)^2} = \sqrt{50}$ $\overline{QR}$ $\sqrt{(1-8)^2 + (4-3)^2} = \sqrt{50}$ $\overline{RS}$ $\sqrt{(-4-1)^2 + (-1-4)^2} = \sqrt{50}$
$\overline{PS}$ $\sqrt{(-4-3)^2 + (-1--2)^2} = \sqrt{50}$ *PQRS* is a rhombus because all sides are congruent. $m_{\overline{PQ}} = \dfrac{8-3}{3--2} = \dfrac{5}{5} = 1$
$m_{\overline{QR}} = \dfrac{1-8}{4-3} = -7$ Because the slopes of adjacent sides are not opposite reciprocals, they are not perpendicular



and do not form a right angle. Therefore *PQRS* is not a square.

REF: 061735geo

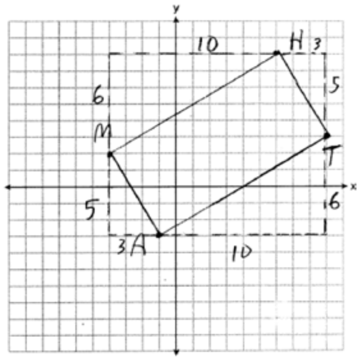
12 ANS:



$\overline{AD}$ and $\overline{BC}$ have equal slope, so are parallel. $\overline{AB}$ and $\overline{CD}$ have equal slope, so are parallel. Since both pairs of opposite sides are parallel, *ABCD* is a parallelogram. The slope of $\overline{AB}$ and $\overline{BC}$ are not opposite reciprocals, so they are not perpendicular, and so $\angle B$ is not a right angle. *ABCD* is not a rectangle since all four angles are not right angles.

REF: 082334geo

13 ANS:

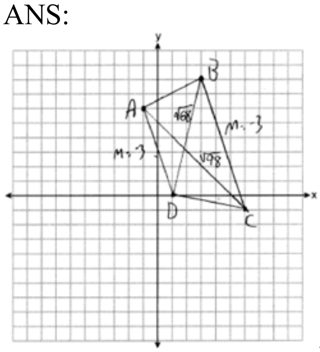


$m_{\overline{MH}} = \frac{6}{10} = \frac{3}{5}$, $m_{\overline{AT}} = \frac{6}{10} = \frac{3}{5}$, $m_{\overline{MA}} = -\frac{5}{3}$, $m_{\overline{HT}} = -\frac{5}{3}$; $\overline{MH} \parallel \overline{AT}$ and $\overline{MA} \parallel \overline{HT}$.

MATH is a parallelogram since both sides of opposite sides are parallel. $m_{\overline{MA}} = -\frac{5}{3}$, $m_{\overline{AT}} = \frac{3}{5}$. Since the slopes are negative reciprocals, $\overline{MA} \perp \overline{AT}$ and $\angle A$ is a right angle. *MATH* is a rectangle because it is a parallelogram with a right angle.

REF: 081835geo

14 ANS:



$m_{\overline{AD}} = \frac{0-6}{1--1} = -3$ $\overline{AD} \parallel \overline{BC}$ because their slopes are equal. *ABCD* is a trapezoid

$m_{\overline{BC}} = \frac{-1-8}{6-3} = -3$

because it has a pair of parallel sides. $AC = \sqrt{(-1-6)^2 + (6--1)^2} = \sqrt{98}$ *ABCD* is not an isosceles trapezoid

$BD = \sqrt{(8-0)^2 + (3-1)^2} = \sqrt{68}$

because its diagonals are not congruent.

REF: 061932geo

15 ANS:

$m_{\overline{AB}} = \frac{6-3}{-3-6} = \frac{3}{-9} = -\frac{1}{3}$ $m_{\overline{BC}} = \frac{3--2}{6-6} = \frac{5}{0} \rightarrow$ undefined *ABCD* is a trapezoid because it has only one pair of

$m_{\overline{CD}} = \frac{2--2}{-6-6} = \frac{4}{-12} = -\frac{1}{3}$ $m_{\overline{AD}} = \frac{6-2}{-3--6} = \frac{4}{3}$

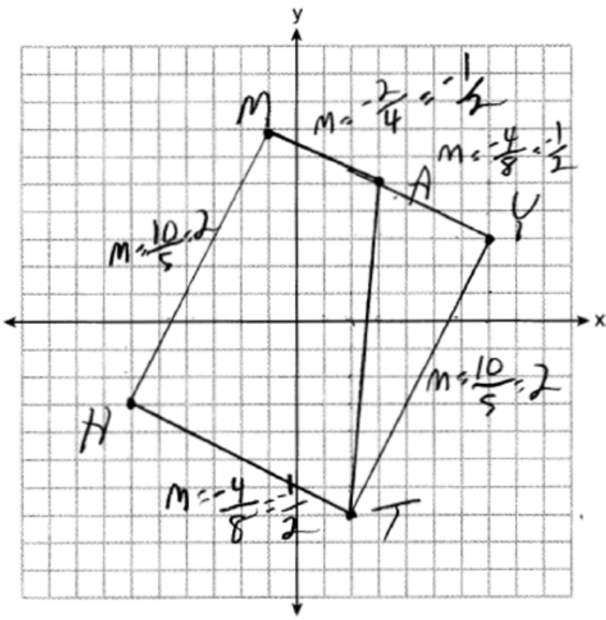
parallel sides. $BD = \sqrt{(6--6)^2 + (3-2)^2} = \sqrt{145}$ *ABCD* is isosceles because *ABCD*'s diagonals are

$AC = \sqrt{(6--3)^2 + (-2-6)^2} = \sqrt{145}$

congruent.

REF: 082433geo

16 ANS:

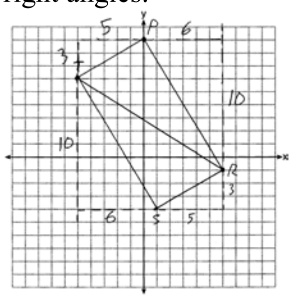


The slope of $\overline{MA}$ and $\overline{TH}$ equals $-\frac{1}{2}$. Distinct lines with equal slope are parallel. *MATH* is a trapezoid because it has a pair of parallel lines. (7,3). The slope of $\overline{MY}$ and $\overline{TH}$ equals $-\frac{1}{2}$. The slope of $\overline{YT}$ and $\overline{HM}$ equals 2. The slopes of each side are opposite reciprocals and therefore perpendicular. Perpendicular sides form right angles, so *MYTH* has four right angles and is a rectangle.

REF: 012435geo

17 ANS:

$m_{\overline{TS}} = \frac{-10}{6} = -\frac{5}{3}$ $m_{\overline{SR}} = \frac{3}{5}$ Since the slopes of $\overline{TS}$ and $\overline{SR}$ are opposite reciprocals, they are perpendicular and form a right angle. $\triangle RST$ is a right triangle because $\angle S$ is a right angle. $P(0,9)$ $m_{\overline{RP}} = \frac{-10}{6} = -\frac{5}{3}$ $m_{\overline{PT}} = \frac{3}{5}$ Since the slopes of all four adjacent sides ($\overline{TS}$ and $\overline{SR}$, $\overline{SR}$ and $\overline{RP}$, $\overline{PT}$ and $\overline{TS}$, $\overline{RP}$ and $\overline{PT}$) are opposite reciprocals, they are perpendicular and form right angles. Quadrilateral $RSTP$ is a rectangle because it has four right angles.

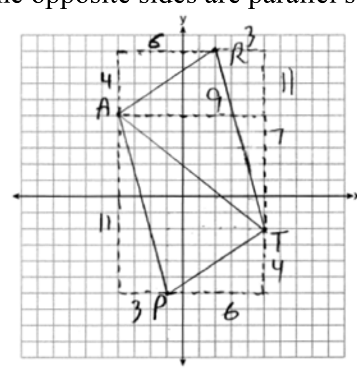


REF: 061536geo

18 ANS:

$\triangle PAT$ is an isosceles triangle because sides $\overline{AP}$ and $\overline{AT}$ are congruent ($\sqrt{3^2 + 11^2} = \sqrt{7^2 + 9^2} = \sqrt{130}$). $R(2,9)$. Quadrilateral $PART$ is a parallelogram because the opposite sides are parallel since they have equal slopes

$(m_{\overline{AR}} = \frac{4}{6} = \frac{2}{3};\ m_{\overline{PT}} = \frac{4}{6} = \frac{2}{3};\ m_{\overline{PA}} = -\frac{11}{3};\ m_{\overline{RT}} = -\frac{11}{3})$

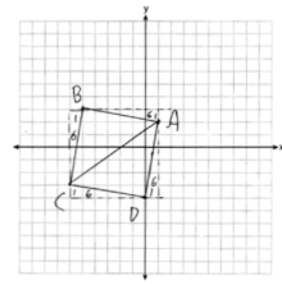


REF: 011835geo

19 ANS:

$AB=\sqrt{(-5-1)^2+(3-2)^2}=\sqrt{37}$, $BC=\sqrt{(-5--6)^2+(3--3)^2}=\sqrt{37}$ (because $AB=BC$, $\triangle ABC$ is isosceles). $(0,-4)$. $AD=\sqrt{(1-0)^2+(2--4)^2}=\sqrt{37}$, $CD=\sqrt{(-6-0)^2+(-3--4)^2}=\sqrt{37}$,

$m_{\overline{AB}}=\dfrac{3-2}{-5-1}=-\dfrac{1}{6}$, $m_{\overline{CB}}=\dfrac{3--3}{-5--6}=6$ ($ABCD$ is a square because all four sides are congruent, consecutive sides are perpendicular since slopes are opposite reciprocals and so $\angle B$ is a right angle).



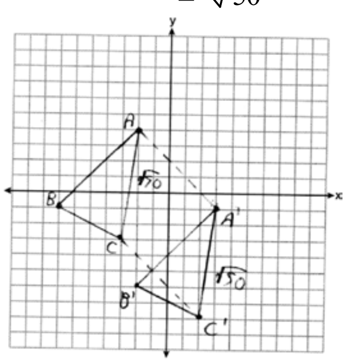
REF: 081935geo

20 ANS:

$\sqrt{(-2--7)^2+(4--1)^2}=\sqrt{(-2--3)^2+(4--3)^2}$ Since $\overline{AB}$ and $\overline{AC}$ are congruent, $\triangle ABC$ is isosceles.

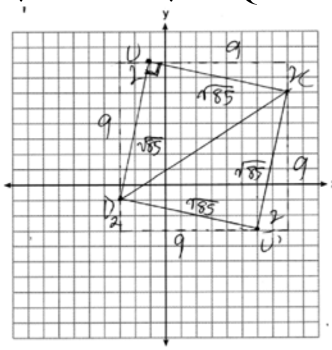
$\sqrt{50}=\sqrt{50}$

$A'(3,-1)$, $B'(-2,-6)$, $C'(2,-8)$. $AC=\sqrt{50}$ $AA'=\sqrt{(-2-3)^2+(4--1)^2}=\sqrt{50}$, $A'C'=\sqrt{50}$ (translation preserves distance), $CC'=\sqrt{(-3-2)^2+(-3--8)^2}=\sqrt{50}$ Since all four sides are congruent, $AA'C'C$ is a rhombus.



REF: 062235geo

21 ANS:
$m_{\overline{DU}} = \frac{9}{2}$ $m_{\overline{UC}} = -\frac{2}{9}$ Since the slopes of $\overline{DU}$ and $\overline{UC}$ are opposite reciprocals, they are perpendicular and form a right angle. $\triangle DUC$ is a right triangle because $\angle DUC$ is a right angle. Each side of quadrilateral $DUCU'$ is $\sqrt{9^2+2^2} = \sqrt{85}$. Quadrilateral $DUCU'$ is a square because all four side are congruent and it has a right angle.



REF: 012335geo

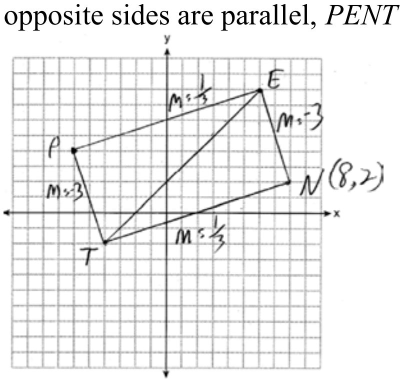
22 ANS:
$m_{\overline{PE}} = \frac{8-4}{6--6} = \frac{4}{12} = \frac{1}{3}$ Since the slopes of $\overline{PE}$ and $\overline{PT}$ are opposite reciprocals, they are perpendicular and

$m_{\overline{PT}} = \frac{4--2}{-6--4} = \frac{6}{-2} = -3$

form a right angle. $\triangle PET$ is a right triangle because it has a right angle. (8,2) $m_{\overline{TN}} = \frac{2--2}{8--4} = \frac{4}{12} = \frac{1}{3}$ Because

$m_{\overline{EN}} = \frac{8-2}{6-8} = \frac{6}{-2} = -3$

the slopes of $\overline{PE}$ and $\overline{TN}$ are equal, $\overline{PE} \parallel \overline{TN}$. Because the slopes of $\overline{PT}$ and $\overline{EN}$ are equal, $\overline{PT} \parallel \overline{EN}$. Because opposite sides are parallel, *PENT* is a parallelogram. Because $\angle P$ is a right angle, *PENT* is a rectangle.



REF: 012535geo

23 ANS:
$M\left(\frac{4+0}{2}, \frac{6-1}{2}\right) = M\left(2, \frac{5}{2}\right)$ $m = \frac{6--1}{4-0} = \frac{7}{4}$ $m_{\perp} = -\frac{4}{7}$ $y - 2.5 = -\frac{4}{7}(x-2)$ The diagonals, $\overline{MT}$ and $\overline{AH}$, of rhombus *MATH* are perpendicular bisectors of each other.

REF: fall1411geo