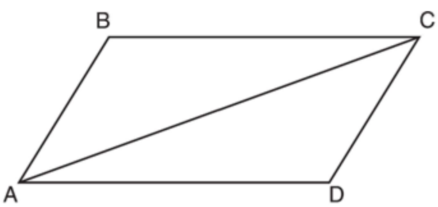
G.SRT.B.5: Quadrilateral Proofs

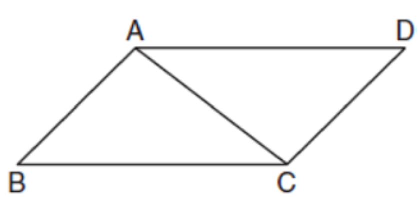
1 Given that *ABCD* is a parallelogram, a student wrote the proof below to show that a pair of its opposite angles are congruent.



Statement	Reason
1. ABCD is a parallelogram.	1. Given
2. $\overline{BC} \cong \overline{AD}$ $\overline{AB} \cong \overline{DC}$	2. Opposite sides of a parallelogram are congruent.
3. $\overline{AC} \cong \overline{CA}$	3. Reflexive Postulate of Congruency
4. $\triangle ABC \cong \triangle CDA$	4. Side-Side-Side
5. $\angle B \cong \angle D$	5. ____________

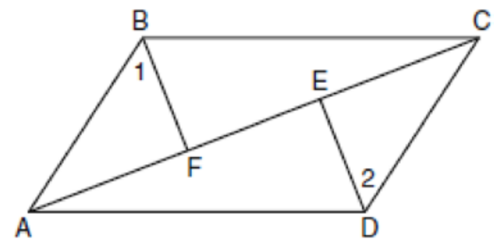
What is the reason justifying that $\angle B \cong \angle D$?
1) Opposite angles in a quadrilateral are congruent.
2) Parallel lines have congruent corresponding angles.
3) Corresponding parts of congruent triangles are congruent.
4) Alternate interior angles in congruent triangles are congruent.

2 Given: Parallelogram *ABCD* with diagonal $\overline{AC}$ drawn

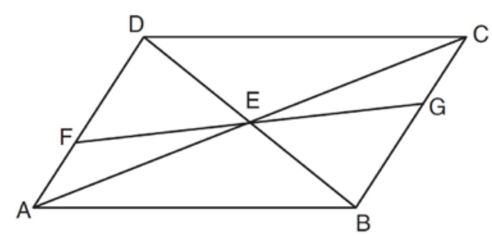


Prove: $\triangle ABC \cong \triangle CDA$

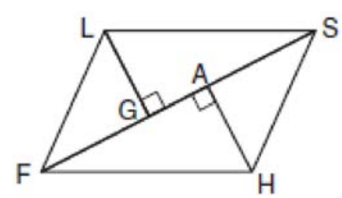
3 Given: Quadrilateral *ABCD*, diagonal $\overline{AFEC}$, $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle 1 \cong \angle 2$
Prove: *ABCD* is a parallelogram.



4 In the diagram below of quadrilateral *ABCD*, $\overline{AD} \cong \overline{BC}$ and $\angle DAE \cong \angle BCE$. Line segments *AC*, *DB*, and *FG* intersect at *E*. Prove: $\triangle AEF \cong \triangle CEG$

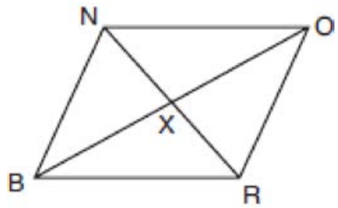


5 Given: parallelogram *FLSH*, diagonal $\overline{FGAS}$, $\overline{LG} \perp \overline{FS}$, $\overline{HA} \perp \overline{FS}$



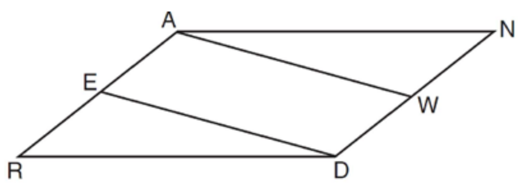
Prove: $\triangle LGS \cong \triangle HAF$

6 The accompanying diagram shows quadrilateral $BRON$, with diagonals $\overline{NR}$ and $\overline{BO}$, which bisect each other at X.



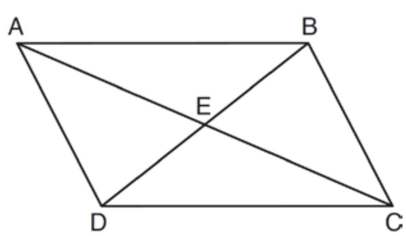
Prove: $\triangle BNX \cong \triangle ORX$

7 Given: Parallelogram $ANDR$ with $\overline{AW}$ and $\overline{DE}$ bisecting $\overline{NWD}$ and $\overline{REA}$ at points W and E, respectively



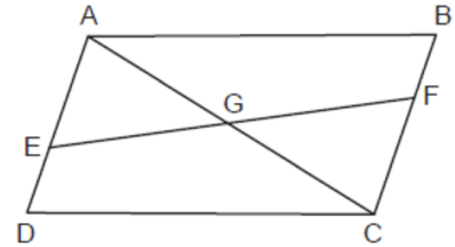
Prove that $\triangle ANW \cong \triangle DRE$. Prove that quadrilateral $AWDE$ is a parallelogram.

8 Given: Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E



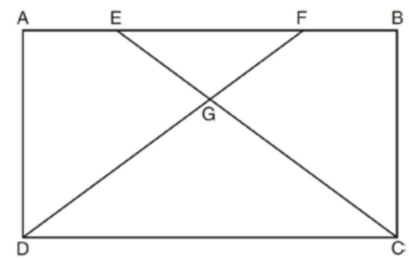
Prove: $\triangle AED \cong \triangle CEB$
Describe a single rigid motion that maps $\triangle AED$ onto $\triangle CEB$.

9 Given: Quadrilateral $ABCD$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$, diagonal $\overline{AC}$ intersects $\overline{EF}$ at G, and $\overline{DE} \cong \overline{BF}$

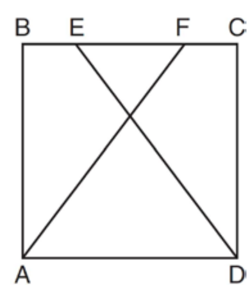


Prove: G is the midpoint of $\overline{EF}$

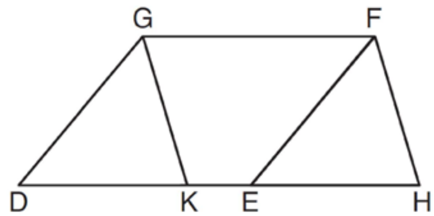
10 The diagram below shows rectangle $ABCD$ with points E and F on side $\overline{AB}$. Segments CE and DF intersect at G, and $\angle ADG \cong \angle BCG$. Prove: $\overline{AE} \cong \overline{BF}$



11 The diagram below shows square $ABCD$ where E and F are points on $\overline{BC}$ such that $\overline{BE} \cong \overline{FC}$, and segments AF and DE are drawn. Prove that $\overline{AF} \cong \overline{DE}$.

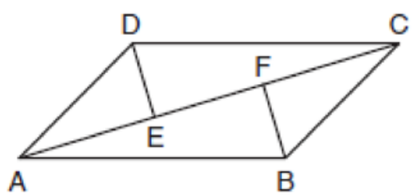


12 Given: Parallelogram *DEFG*, *K* and *H* are points on $\overrightarrow{DE}$ such that $\angle DGK \cong \angle EFH$ and $\overline{GK}$ and $\overline{FH}$ are drawn.



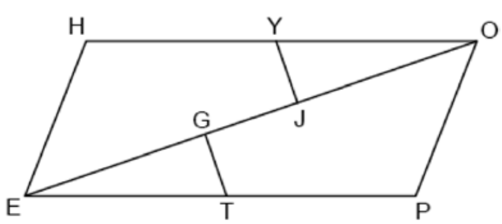
Prove: $\overline{DK} \cong \overline{EH}$

13 In quadrilateral *ABCD*, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$, and $\overline{BF}$ and $\overline{DE}$ are perpendicular to diagonal $\overline{AC}$ at points *F* and *E*.



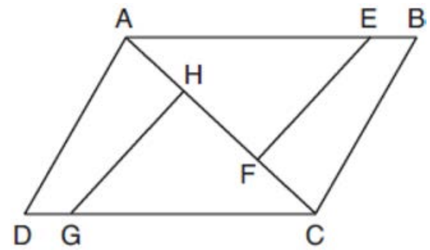
Prove: $\overline{AE} \cong \overline{CF}$

14 In quadrilateral *HOPE* below, $\overline{EH} \cong \overline{OP}$, $\overline{EP} \cong \overline{OH}$, $\overline{EJ} \cong \overline{OG}$, and $\overline{TG}$ and $\overline{YJ}$ are perpendicular to diagonal $\overline{EO}$ at points *G* and *J*, respectively.



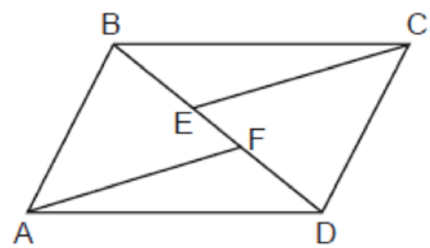
Prove that $\overline{TG} \cong \overline{YJ}$.

15 In the diagram of quadrilateral *ABCD* with diagonal $\overline{AC}$ shown below, segments *GH* and *EF* are drawn, $\overline{AE} \cong \overline{CG}$, $\overline{BE} \cong \overline{DG}$, $\overline{AH} \cong \overline{CF}$, and $\overline{AD} \cong \overline{CB}$.



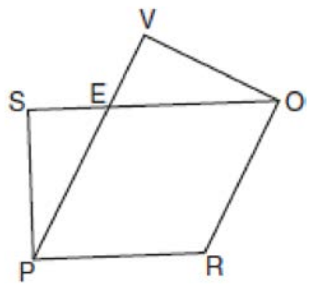
Prove: $\overline{EF} \cong \overline{GH}$

16 In the diagram of quadrilateral *ABCD* below, $\overline{AB} \cong \overline{CD}$, and $\overline{AB} \parallel \overline{CD}$. Segments *CE* and *AF* are drawn to diagonal $\overline{BD}$ such that $\overline{BE} \cong \overline{DF}$.



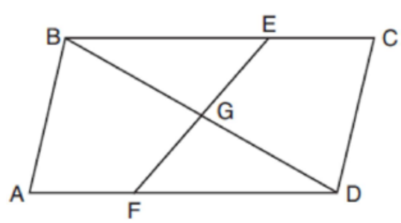
Prove: $\overline{CE} \cong \overline{AF}$

17 Given: *PROE* is a rhombus, $\overline{SEO}$, $\overline{PEV}$, $\angle SPR \cong \angle VOR$



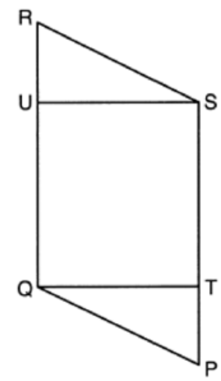
Prove: $\overline{SE} \cong \overline{EV}$

18 In quadrilateral *ABCD*, *E* and *F* are points on $\overline{BC}$ and $\overline{AD}$, respectively, and $\overline{BGD}$ and $\overline{EGF}$ are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$.



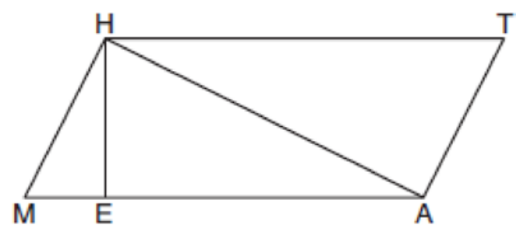
Prove: $\overline{FG} \cong \overline{EG}$

19 Given: Parallelogram *PQRS*, $\overline{QT} \perp \overline{PS}$, $\overline{SU} \perp \overline{QR}$



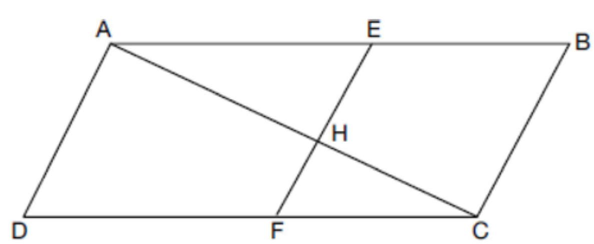
Prove: $\overline{PT} \cong \overline{RU}$

20 Given: Quadrilateral *MATH*, $\overline{HM} \cong \overline{AT}$, $\overline{HT} \cong \overline{AM}$, $\overline{HE} \perp \overline{MEA}$, and $\overline{HA} \perp \overline{AT}$



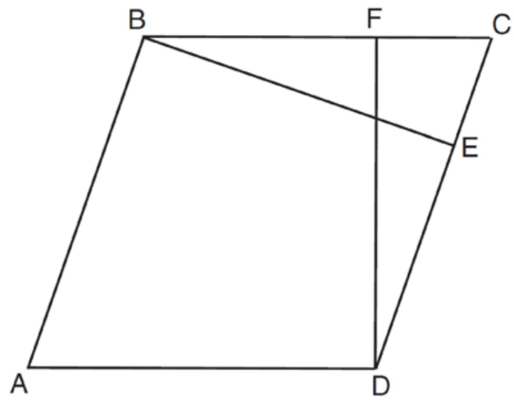
Prove: $TA \bullet HA = HE \bullet TH$

21 Given: Quadrilateral *ABCD*, $\overline{AC}$ and $\overline{EF}$ intersect at *H*, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$.



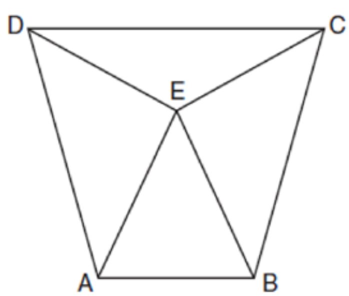
Prove: $(EH)(CH) = (FH)(AH)$

22 In the diagram of parallelogram $ABCD$ below, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, $\overline{CE} \cong \overline{CF}$.



Prove $ABCD$ is a rhombus.

23 Isosceles trapezoid $ABCD$ has bases $\overline{DC}$ and $\overline{AB}$ with nonparallel legs $\overline{AD}$ and $\overline{BC}$. Segments AE, BE, CE, and DE are drawn in trapezoid $ABCD$ such that $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$.



Prove $\triangle ADE \cong \triangle BCE$ and prove $\triangle AEB$ is an isosceles triangle.

24 Given: Quadrilateral $ABCD$ with $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$, and diagonal $\overline{BD}$ is drawn
Prove: $\angle BDC \cong \angle ABD$

25 Prove that the diagonals of a parallelogram bisect each other.

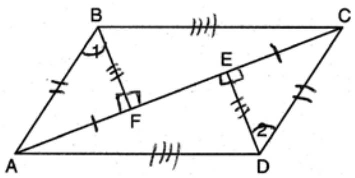
26 A tricolored flag is made out of a rectangular piece of cloth whose corners are labeled A, B, C, and D. The colored regions are separated by two line segments, $\overline{BM}$ and $\overline{CM}$, that meet at point M, the midpoint of side $\overline{AD}$. Prove that the two line segments that separate the regions will always be equal in length, regardless of the size of the flag.

G.SRT.B.5: Quadrilateral Proofs
Answer Section

1 ANS: 3 REF: 081208ge

2 ANS:
Parallelogram $ABCD$ with diagonal $\overline{AC}$ drawn (given). $\overline{AC} \cong \overline{AC}$ (reflexive property). $\overline{AD} \cong \overline{CB}$ and $\overline{BA} \cong \overline{DC}$ (opposite sides of a parallelogram are congruent). $\triangle ABC \cong \triangle CDA$ (SSS).

REF: 011825geo

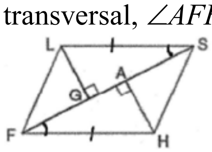
3 ANS:


$\overline{FE} \cong \overline{FE}$ (Reflexive Property); $\overline{AE} - \overline{FE} \cong \overline{FC} - \overline{EF}$ (Line Segment Subtraction Theorem); $\overline{AF} \cong \overline{CE}$ (Substitution); $\angle BFA \cong \angle DEC$ (All right angles are congruent); $\triangle BFA \cong \triangle DEC$ (AAS); $\overline{AB} \cong \overline{CD}$ and $\overline{BF} \cong \overline{DE}$ (CPCTC); $\angle BFC \cong \angle DEA$ (All right angles are congruent); $\triangle BFC \cong \triangle DEA$ (SAS); $\overline{AD} \cong \overline{CB}$ (CPCTC); $ABCD$ is a parallelogram (opposite sides of quadrilateral $ABCD$ are congruent)

REF: 080938ge

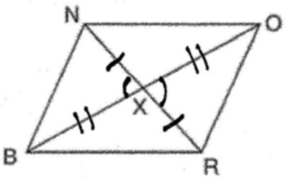
4 ANS:
Quadrilateral $ABCD$, $\overline{AD} \cong \overline{BC}$ and $\angle DAE \cong \angle BCE$ are given. $\overline{AD} \parallel \overline{BC}$ because if two lines are cut by a transversal so that a pair of alternate interior angles are congruent, the lines are parallel. $ABCD$ is a parallelogram because if one pair of opposite sides of a quadrilateral are both congruent and parallel, the quadrilateral is a parallelogram. $\overline{AE} \cong \overline{CE}$ because the diagonals of a parallelogram bisect each other. $\angle FEA \cong \angle GEC$ as vertical angles. $\triangle AEF \cong \triangle CEG$ by ASA.

REF: 011238ge

5 ANS:
Because $FLSH$ is a parallelogram, $\overline{FH} \cong \overline{SL}$. Because $FLSH$ is a parallelogram, $\overline{FH} \parallel \overline{SL}$ and since $\overline{FGAS}$ is a transversal, $\angle AFH$ and $\angle LSG$ are alternate interior angles and congruent. Therefore $\triangle LGS \cong \triangle HAF$ by AAS.


REF: 010634b

6 ANS:
Because diagonals $\overline{NR}$ and $\overline{BO}$ bisect each other, $\overline{NX} \cong \overline{RX}$ and $\overline{BX} \cong \overline{OX}$. $\angle BXN$ and $\angle OXR$ are congruent vertical angles. Therefore $\triangle BNX \cong \triangle ORX$ by SAS.



REF: 080731b

7 ANS:
Parallelogram $ANDR$ with $\overline{AW}$ and $\overline{DE}$ bisecting $\overline{NWD}$ and $\overline{REA}$ at points W and E (Given). $\overline{AN} \cong \overline{RD}$, $\overline{AR} \cong \overline{DN}$ (Opposite sides of a parallelogram are congruent). $AE = \frac{1}{2}AR$, $WD = \frac{1}{2}DN$, so $\overline{AE} \cong \overline{WD}$ (Definition of bisect and division property of equality). $\overline{AR} \parallel \overline{DN}$ (Opposite sides of a parallelogram are parallel). $AWDE$ is a parallelogram (Definition of parallelogram). $RE = \frac{1}{2}AR$, $NW = \frac{1}{2}DN$, so $\overline{RE} \cong \overline{NW}$ (Definition of bisect and division property of equality). $\overline{ED} \cong \overline{AW}$ (Opposite sides of a parallelogram are congruent). $\triangle ANW \cong \triangle DRE$ (SSS).

REF: 011635geo

8 ANS:
Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E (Given). $\overline{AD} \cong \overline{BC}$ (Opposite sides of a parallelogram are congruent). $\angle AED \cong \angle CEB$ (Vertical angles are congruent). $\overline{BC} \parallel \overline{DA}$ (Definition of parallelogram). $\angle DBC \cong \angle BDA$ (Alternate interior angles are congruent). $\triangle AED \cong \triangle CEB$ (AAS). 180° rotation of $\triangle AED$ around point E.

REF: 061533geo

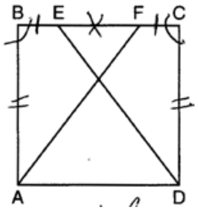
9 ANS:
Quadrilateral $ABCD$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$, diagonal $\overline{AC}$ intersects $\overline{EF}$ at G, and $\overline{DE} \cong \overline{BF}$ (given); $ABCD$ is a parallelogram (a quadrilateral with a pair of opposite sides $\parallel$ is a parallelogram); $\overline{AD} \cong \overline{CB}$ (opposite side of a parallelogram are congruent); $\overline{AE} \cong \overline{CF}$ (subtraction postulate); $\overline{AD} \parallel \overline{CB}$ (opposite side of a parallelogram are parallel); $\angle EAG \cong \angle FCG$ (if parallel sides are cut by a transversal, the alternate interior angles are congruent); $\angle AGE \cong \angle CGF$ (vertical angles); $\triangle AEG \cong \triangle CFG$ (AAS); $\overline{EG} \cong \overline{FG}$ (CPCTC): G is the midpoint of $\overline{EF}$ (since G divides $\overline{EF}$ into two equal parts, G is the midpoint of $\overline{EF}$).

REF: 062335geo

10 ANS:
Rectangle $ABCD$ with points E and F on side $\overline{AB}$, segments CE and DF intersect at G, and $\angle ADG \cong \angle BCE$ are given. $\overline{AD} \cong \overline{BC}$ because opposite sides of a rectangle are congruent. $\angle A$ and $\angle B$ are right angles and congruent because all angles of a rectangle are right and congruent. $\triangle ADF \cong \triangle BCE$ by ASA. $\overline{AF} \cong \overline{BE}$ per CPCTC. $\overline{EF} \cong \overline{FE}$ under the Reflexive Property. $\overline{AF} - \overline{EF} \cong \overline{BE} - \overline{FE}$ using the Subtraction Property of Segments. $\overline{AE} \cong \overline{BF}$ because of the Definition of Segments.

REF: 011338ge

11 ANS:



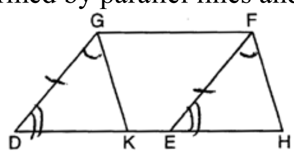
Square $ABCD$; E and F are points on $\overline{BC}$ such that $\overline{BE} \cong \overline{FC}$; $\overline{AF}$ and $\overline{DE}$ drawn (Given). $\overline{AB} \cong \overline{CD}$ (All sides of a square are congruent). $\angle ABF \cong \angle DCE$ (All angles of a square are equiangular). $\overline{EF} \cong \overline{FE}$ (Reflexive property). $\overline{BE} + \overline{EF} \cong \overline{FC} + \overline{FE}$ (Additive property of line segments). $\overline{BF} \cong \overline{CE}$ (Angle addition). $\triangle ABF \cong \triangle DCE$ (SAS). $\overline{AF} \cong \overline{DE}$ (CPCTC).

REF: 061538ge

12 ANS:

Parallelogram $DEFG$, K and H are points on $\overrightarrow{DE}$ such that $\angle DGK \cong \angle EFH$ and $\overline{GK}$ and $\overline{FH}$ are drawn (given). $\overline{DG} \cong \overline{EF}$ (opposite sides of a parallelogram are congruent). $\overline{DG} \parallel \overline{EF}$ (opposite sides of a parallelogram are parallel). $\angle D \cong \angle FEH$ (corresponding angles formed by parallel lines and a transversal are congruent).

$\triangle DGK \cong \triangle EFH$ (ASA). $\overline{DK} \cong \overline{EH}$ (CPCTC).



REF: 081538ge

13 ANS:

Quadrilateral $ABCD$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$, and $\overline{BF}$ and $\overline{DE}$ are perpendicular to diagonal $\overline{AC}$ at points F and E (given). $\angle AED$ and $\angle CFB$ are right angles (perpendicular lines form right angles). $\angle AED \cong \angle CFB$ (All right angles are congruent). $ABCD$ is a parallelogram (A quadrilateral with one pair of sides congruent and parallel is a parallelogram). $\overline{AD} \parallel \overline{BC}$ (Opposite sides of a parallelogram are parallel). $\angle DAE \cong \angle BCF$ (Parallel lines cut by a transversal form congruent alternate interior angles). $\overline{DA} \cong \overline{BC}$ (Opposite sides of a parallelogram are congruent). $\triangle ADE \cong \triangle CBF$ (AAS). $\overline{AE} \cong \overline{CF}$ (CPCTC).

REF: 011735geo

14 ANS:

Quad $HOPE$, $\overline{EH} \cong \overline{OP}$, $\overline{EP} \cong \overline{OH}$, $\overline{EJ} \cong \overline{OG}$, $\overline{TG} \perp \overline{EO}$ and $\overline{YJ} \perp \overline{EO}$ (Given); $HOPE$ is a parallelogram (Both pairs of opposite sides are parallel); $\overline{HO} \parallel \overline{PE}$ (Opposite sides of a parallelogram are parallel); $\angle YOJ \cong \angle GET$ (Parallel lines cut by a transversal form congruent alternate interior angles); $\overline{GJ} \cong \overline{GJ}$ (Reflexive); $\overline{EG} \cong \overline{OJ}$ (Subtraction); $\angle EGT$ and $\angle OJY$ are right angles (Perpendicular lines form right angles); $\angle EGT \cong \angle OJY$ (All right angles are congruent); $\triangle EGT \cong \triangle OJY$ (ASA); $\overline{TG} \cong \overline{YJ}$ (CPCTC).

REF: 082435geo

15 ANS:

Quadrilateral $ABCD$ with diagonal $\overline{AC}$, segments GH and EF, $\overline{AE} \cong \overline{CG}$, $\overline{BE} \cong \overline{DG}$, $\overline{AH} \cong \overline{CF}$, and $\overline{AD} \cong \overline{CB}$ (given); $\overline{HF} \cong \overline{HF}$, $\overline{AC} \cong \overline{AC}$ (reflexive property); $\overline{AH} + \overline{HF} \cong \overline{CF} + \overline{HF}$, $\overline{AE} + \overline{BE} \cong \overline{CG} + \overline{DG}$ (segment

$\overline{AF} \cong \overline{CH}$ $\overline{AB} \cong \overline{CD}$

addition); $\triangle ABC \cong \triangle CDA$ (SSS); $\angle EAF \cong \angle GCH$ (CPCTC); $\triangle AEF \cong \triangle CGH$ (SAS); $\overline{EF} \cong \overline{GH}$ (CPCTC).

REF: 011935geo

16 ANS:

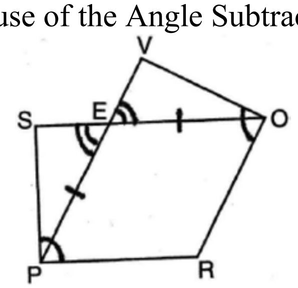
In quadrilateral $ABCD$, $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$, segments CE and AF are drawn to diagonal $\overline{BD}$ such that $\overline{BE} \cong \overline{DF}$ (Given); $\angle ABF \cong \angle CDE$ (Parallel lines cut by a transversal form congruent interior angles); $\overline{EF} \cong \overline{FE}$ (Reflexive); $\overline{BE} + \overline{EF} \cong \overline{DF} + \overline{FE}$ (Addition); $\triangle AFB \cong \triangle CED$ (SAS); $\overline{CE} \cong \overline{AF}$ *(CPCTC).*

$\overline{BF} \cong \overline{DE}$

REF: 012434geo

17 ANS:

Because $PROE$ is a rhombus, $\overline{PE} \cong \overline{OE}$. $\angle SEP \cong \angle VEO$ are congruent vertical angles. $\angle EPR \cong \angle EOR$ because opposite angles of a rhombus are congruent. $\angle SPE \cong \angle VOE$ because of the Angle Subtraction Theorem. $\triangle SEP \cong \triangle VEO$ because of ASA. $\overline{SE} \cong \overline{EV}$ because of CPCTC.



REF: 010934b

18 ANS:

Quadrilateral $ABCD$, E and F are points on $\overline{BC}$ and $\overline{AD}$, respectively, and $\overline{BGD}$ and $\overline{EGF}$ are drawn such that $\angle ABG \cong \angle CDG$, $\overline{AB} \cong \overline{CD}$, and $\overline{CE} \cong \overline{AF}$ (given); $\overline{BD} \cong \overline{BD}$ (reflexive); $\triangle ABD \cong \triangle CDB$ (SAS); $\overline{BC} \cong \overline{DA}$ (CPCTC); $\overline{BE} + \overline{CE} \cong \overline{AF} + \overline{DF}$ (segment addition); $\overline{BE} \cong \overline{DF}$ (segment subtraction); $\angle BGE \cong \angle DGF$ (vertical angles are congruent); $\angle CBD \cong \angle ADB$ (CPCTC); $\triangle EBG \cong \triangle FDG$ (AAS); $\overline{FG} \cong \overline{EG}$ (CPCTC).

REF: 012035geo

19 ANS:

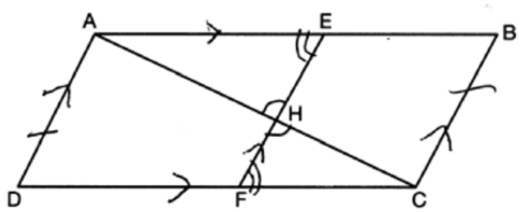
Parallelogram $PQRS$, $\overline{QT} \perp \overline{PS}$, $\overline{SU} \perp \overline{QR}$ (given); $\overline{QUR} \cong \overline{PTS}$ (opposite sides of a parallelogram are parallel; Quadrilateral $QUST$ is a rectangle (quadrilateral with parallel opposite sides and opposite right angles is a rectangle); $\overline{SU} \cong \overline{QT}$ (opposite sides of a rectangle are congruent); $\overline{RS} \cong \overline{PQ}$ (opposite sides of a parallelogram are congruent); $\angle RUS$ and $\angle PTQ$ are right angles (the supplement of a right angle is a right angle), $\triangle RSU \cong \triangle PQT$ (HL); $\overline{PT} \cong \overline{RU}$ (CPCTC)

REF: 062233geo

20 ANS:
Quadrilateral *MATH*, $\overline{HM} \cong \overline{AT}$, $\overline{HT} \cong \overline{AM}$, $\overline{HE} \perp \overline{MEA}$, and $\overline{HA} \perp \overline{AT}$ (given); $\angle HEA$ and $\angle TAH$ are right angles (perpendicular lines form right angles); $\angle HEA \cong \angle TAH$ (all right angles are congruent); *MATH* is a parallelogram (a quadrilateral with two pairs of congruent opposite sides is a parallelogram); $\overline{MA} \parallel \overline{TH}$ (opposite sides of a parallelogram are parallel); $\angle THA \cong \angle EAH$ (alternate interior angles of parallel lines and a transversal are congruent); $\triangle HEA \sim \triangle TAH$ (AA); $\frac{HA}{TH} = \frac{HE}{TA}$ (corresponding sides of similar triangles are in proportion); $TA \bullet HA = HE \bullet TH$ (product of means equals product of extremes).

REF: 061935geo

21 ANS:



1) Quadrilateral *ABCD*, $\overline{AC}$ and $\overline{EF}$ intersect at *H*, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$ (Given); 2) $\angle EHA \cong \angle FHC$ (Vertical angles are congruent); 3) $\overline{AD} \parallel \overline{BC}$ (Transitive property of parallel lines); 4) *ABCD* is a parallelogram (Quadrilateral with a pair of sides both parallel and congruent); 5) $\overline{AB} \parallel \overline{CD}$ (Opposite sides of a parallelogram); 6) $\angle AEH \cong \angle CFH$ (Alternate interior angles formed by parallel lines and a transversal); 7) $\triangle AEH \sim \triangle CFH$ (AA); 8) $\frac{EH}{FH} = \frac{AH}{CH}$ (Corresponding sides of similar triangles are proportional); 8) $(EH)(CH) = (FH)(AH)$ (Product of means equals product of extremes).

REF: 082235geo

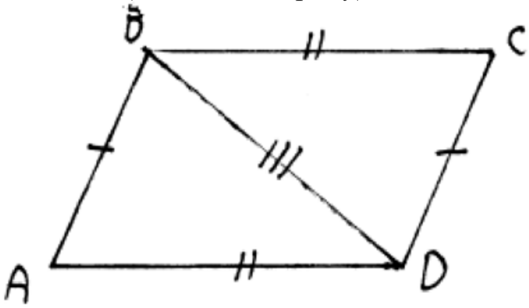
22 ANS:
Parallelogram *ABCD*, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, $\overline{CE} \cong \overline{CF}$ (given). $\angle BEC \cong \angle DFC$ (perpendicular lines form right angles, which are congruent). $\angle FCD \cong \angle BCE$ (reflexive property). $\triangle BEC \cong \triangle DFC$ (ASA). $\overline{BC} \cong \overline{CD}$ (CPCTC). *ABCD* is a rhombus (a parallelogram with consecutive congruent sides is a rhombus).

REF: 081535geo

23 ANS:
Isosceles trapezoid *ABCD*, $\angle CDE \cong \angle DCE$, $\overline{AE} \perp \overline{DE}$, and $\overline{BE} \perp \overline{CE}$ (given); $\overline{AD} \cong \overline{BC}$ (congruent legs of isosceles trapezoid); $\angle DEA$ and $\angle CEB$ are right angles (perpendicular lines form right angles); $\angle DEA \cong \angle CEB$ (all right angles are congruent); $\angle CDA \cong \angle DCB$ (base angles of an isosceles trapezoid are congruent); $\angle CDA - \angle CDE \cong \angle DCB - \angle DCE$ (subtraction postulate); $\triangle ADE \cong \triangle BCE$ (AAS); $\overline{EA} \cong \overline{EB}$ (CPCTC);
$\angle EDA \cong \angle ECB$
$\triangle AEB$ is an isosceles triangle (an isosceles triangle has two congruent sides).

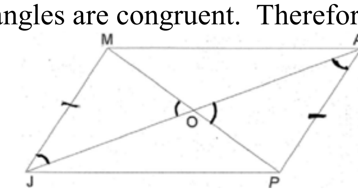
REF: 081735geo

24 ANS:
$\overline{BD} \cong \overline{DB}$ (Reflexive Property); $\triangle ABD \cong \triangle CDB$ (SSS); $\angle BDC \cong \angle ABD$ (CPCTC).



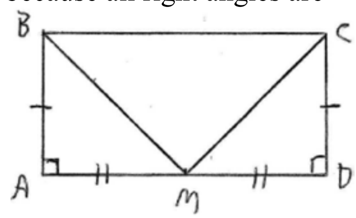
REF: 061035ge

25 ANS:
Assume parallelogram $JMAP$ with diagonals intersecting at O. Opposite sides of a parallelogram are congruent, so $\overline{JM} \cong \overline{AP}$. $\angle JOM$ and $\angle AOP$ are congruent vertical angles. Because $JMAP$ is a parallelogram, $\overline{JM} \parallel \overline{AP}$ and since $\overline{JOA}$ is a transversal, $\angle MJO$ and $\angle PAO$ are alternate interior angles and congruent. Therefore $\triangle MJO \cong \triangle PAO$ by AAS. Corresponding parts of congruent triangles are congruent. Therefore $\overline{JO} \cong \overline{AO}$ and $\overline{MO} \cong \overline{PO}$ and the diagonals of a parallelogram bisect each other.



REF: 010233b

26 ANS:
$\overline{AB} \cong \overline{CD}$, because opposite sides of a rectangle are congruent. $\overline{AM} \cong \overline{DM}$, because of the definition of midpoint. $\angle A$ and $\angle D$ are right angles because a rectangle has four right angles. $\angle A \cong \angle D$, because all right angles are congruent. $\triangle ABM \cong \triangle DCM$, because of SAS. $\overline{BM} \cong \overline{CM}$ because of CPCTC.



REF: 080834b